Profit Maximizing Prior-free Multi-unit Procurement Auctions with Capacitated Sellers

(Extended Abstract)

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ABSTRACT

In this paper, we derive bounds for profit maximizing prior-free procurement auctions where a buyer wishes to procure multiple units of a homogeneous item from n sellers who are strategic about their per unit valuation. The buyer earns the profit by reselling these units in an external consumer market. The paper looks at three scenarios of increasing complexity. First, we look at unit capacity sellers where per unit valuation is private information of each seller and the revenue curve is concave. For this setting, we define two benchmarks. We show that no randomized prior free auction can be constant competitive against any of these two benchmarks. However, for a lightly constrained benchmark we design a prior-free auction PEPA (Profit Extracting Procurement Auction) which is 4-competitive and we show this bound is tight. Second, we study a setting where the sellers have non-unit capacities that are common knowledge and derive similar results. In particular, we propose a prior free auction PEPAC (Profit Extracting Procurement Auction with Capacity) which is truthful for any concave revenue curve. Third, we obtain results in the inherently harder bi-dimensional case where per unit valuation as well as capacities are private information of the sellers. We show that PEPAC is truthful and constant competitive for linear revenue curves. We believe that this paper represents the first set of results on single dimensional and bi-dimensional profit maximizing prior-free multi-unit procurement auctions.

Categories and Subject Descriptors

J.4 [**Social and Behavioral Sciences**]: Economics

General Terms

Economics, Theory

Keywords

Procurement; Prior-free; Profit; Auction Design

1. INTRODUCTION

Procurement auctions for awarding contracts to supply goods or services are prevalent in many modern resource allocation situations. In several of these scenarios, the buyer plays the role of an intermediary who purchases some goods or services from the suppliers and resells it in the consumer market. For example, in the retail sector, an intermediary procures products from different vendors (perhaps through an auction) and resells it in consumer markets for a profit. Solving such problems via optimal auction of the kind discussed in the auction literature [3] inevitably requires assumption of a *prior* distribution on the sellers' valuations. The requirement of a known prior distribution often places severe practical limitations. This motivates us to study *prior-free* auctions. In particular, in this paper, we study profit maximizing prior-free procurement auctions with one buyer and n sellers.

The problem of designing a revenue-optimal auction was first studied by Myerson [3]. Iyengar and Kumar [2] consider the problem of designing an optimal procurement auction where the buyer purchases multiple units of a single item from the suppliers and resells it in the consumer market to earn some profit. We consider the same setting here, however, we focus on the design of *prior-free* auctions unlike the *prior-dependent* optimal auction designed in [2]. Goldberg et al. [1] initiated work on design of prior-free auctions and studied a class of single-round sealed-bid auctions for an item in unlimited supply, such as digital goods where each bidder requires at most one unit. They introduced the notion of *competitive* auctions and proposed prior-free randomized competitive auctions based on random sampling.

Although the design of prior-free auctions has generated wide interest in the research community most of the works have considered the forward setting. The reverse auction setting is subtly different from forward auctions especially if the sellers are capacitated and the techniques used for forward auctions cannot be trivially extended to the case of procurement auctions. Moreover, the existing literature on prior-free auctions is limited to the single-dimensional setting where each bidder has only one private type. However, in a procurement auction, the sellers are often capacitated and strategically report their capacities to increase their utilities. Therefore, the design of bi-dimensional prior-free procurement auctions is extremely relevant in practice and in this paper, we believe we have derived the first set of results in this direction.

2. SELLERS WITH UNIT CAPACITIES

First, we consider a single round procurement auction setting with one buyer and n sellers where each seller has a single unit of a homogeneous item. The buyer procures multiple

Appears in: *Proceedings of the 14th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2015), Bordini, Elkind, Weiss, Yolum (eds.), May 4–8, 2015, Istanbul, Turkey.*

units from the sellers and subsequently resells it in an outside consumer market, earning a revenue of $\mathcal{R}(q)$ from selling q units of the item. We assume that the revenue curve of the outside market $\mathcal{R}(q)$ is concave with $\mathcal{R}(0) = 0$. Each bidder (seller) has a private valuation v_i which represents the true minimum amount he is willing to receive to sell a single unit of the item. Given the bid vector $\mathbf{b} = (b_1, \dots, b_n)$ and the revenue curve $\mathcal{R}$, the auctioneer (buyer) computes an allocation $\mathbf{x} = (x_1, \dots, x_n)$, and payments $\mathbf{p} = (p_1, \dots, p_n)$. The profit of the auction (or auctioneer) is given by,

$$\mathcal{A}(\mathbf{b}, \mathcal{R}) = \mathcal{R}(\sum_{i=1}^{n} x_i(\mathbf{b}, \mathcal{R})) - \sum_{i=1}^{n} p_i(\mathbf{b}, \mathcal{R}).$$

The auctioneer wishes to maximize her profit satisfying IR (Individual Rationality) and DSIC (Dominant Strategy Incentive Compatibility).

Prior-Free Benchmarks

As a first step in comparing the performance of any prior-free procurement auction, we need to come up with the right metric for comparison that is a benchmark. It is important that we choose such a benchmark carefully for such a comparison to be meaningful. Here, we start with the strongest possible benchmark for comparison: the profit of an auctioneer who knows the bidder's true valuations. This leads us to consider the two most natural metrics for comparison – the optimal multiple price auction ($\mathcal{T}$) and optimal single price auction ($\mathcal{F}$). We compare the performances of truthful auctions to that of the optimal multiple price and single price auctions. Let $v_{[i]}$ denote the i-th lowest valuation. The profit of $\mathcal{T}$ and $\mathcal{F}$ is given by,

$$\mathcal{T}(\mathbf{b}, \mathcal{R}) = \max_{0 \le i \le n} (\mathcal{R}(i) - \sum_{j=1}^{i} v_{[j]}).$$

$$\mathcal{F}(\mathbf{b}, \mathcal{R}) = \max_{0 \le i \le n} (\mathcal{R}(i) - i v_{[i]}).$$

It is clear that $\mathcal{T}(\mathbf{b}, \mathcal{R}) \ge \mathcal{F}(\mathbf{b}, \mathcal{R})$ for any bid vector $\mathbf{b}$ and any revenue curve $\mathcal{R}$. However, $\mathcal{F}$ does not perform very poorly compared to $\mathcal{T}$. We prove a bound between the performance of $\mathcal{F}$ and $\mathcal{T}$. Specifically, we observe that in the worst case, the maximum ratio of $\mathcal{T}$ to $\mathcal{F}$ is logarithmic in the number n of bidders.

Impossibility results against the Benchmarks

We show that no truthful auction can be constant-competitive against $\mathcal{F}$ and hence it cannot be competitive against $\mathcal{T}$. Specifically, we cannot match the performance of the optimal single price auction when the optimal profit is generated from the single lowest bid. Therefore we present an auction that is *competitive* against $\mathcal{F}^{(2)}$, the optimal single price auction that buys at least two units. Such an auction achieves a constant fraction of the profit of $\mathcal{F}^{(2)}$ on *all inputs*.

Profit Extracting Procurement Auction (PEPA)

We now present a prior-free procurement auction based on random sampling. We extend the *profit extraction* technique of [1]. The goal of the technique is, given $\mathbf{b}$, $\mathcal{R}$, and profit P, to find a subset of bidders who generate profit P.

Profit Extraction ($PE_P(\mathbf{b}, \mathcal{R})$)

Given target profit P,

1. Find the largest value of k for which $v_{[k]}$ is at most $(\mathcal{R}(k) - P)/k$.
2. Pay these k bidders $(\mathcal{R}(k) - P)/k$ and reject others.

Profit Extracting Procurement Auction (PEPA)

1. Partition the bids $\mathbf{b}$ uniformly at random into two sets $\mathbf{b}'$ and $\mathbf{b}''$: for each bid, flip a fair coin, and with probability $1/2$ put the bid in $\mathbf{b}'$ and otherwise in $\mathbf{b}''$.
2. Compute $F' = \mathcal{F}(\mathbf{b}', \mathcal{R})$ and $F'' = \mathcal{F}(\mathbf{b}'', \mathcal{R})$ which are the optimal single price profits for $\mathbf{b}'$ and $\mathbf{b}''$ respectively.
3. Compute the auction results of $\mathrm{PE}_{F''}(\mathbf{b}', \mathcal{R})$ and $\mathrm{PE}_{F'}(\mathbf{b}'', \mathcal{R})$.
4. Run the auction $\mathrm{PE}_{F''}(\mathbf{b}', \mathcal{R})$ or $\mathrm{PE}_{F'}(\mathbf{b}'', \mathcal{R})$ that gives higher profit to the buyer. Ties are broken arbitrarily.

Theorem 1. *PEPA is* 4*-competitive against* $\mathcal{F}^{(2)}$ *for any concave revenue curve* $\mathcal{R}$ *and this bound is tight.*

3. SELLERS WITH NON-UNIT CAPACITIES

Second, we consider the setting where sellers can supply more than one unit of an item. Seller i has valuation per unit v_i and a maximum capacity q_i where v_i is a positive real number and q_i is a positive integer. In other words, each seller can supply at most q_i units of a homogeneous item. We assume that the sellers are strategic with respect to valuation per unit only and they always report their capacities truthfully. We extend the prior-free benchmarks and profit extraction technique for this setting and design Profit Extracting Procurement Auction with Capacity (PEPAC).

Theorem 2. *PEPAC is* $4 \cdot \left(\frac{q_{\max}}{q_{\min}}\right)$*-competitive for any concave revenue curve* $\mathcal{R}$ *if* $q_i \in [q_{\min}, q_{\max}]$ $\forall\, i \in \{1, \dots, n\}$.

Third, we consider bi-dimensional sellers where seller i can misreport his capacity q_i in addition to misreporting his valuation per unit v_i to maximize his gain from the auction. Here, we assume that sellers are not allowed to overbid their capacity. This can be enforced by declaring, as part of the auction, that if a seller fails to provide the number of units he has bid, he suffers a huge penalty (financial or legal loss). But underbidding may help a seller as depending on the mechanism it may result in an increase in the payment which can often more than compensate the loss due to a decrease in allocation. We show that the previous bound of PEPAC holds for the specific case of linear revenue curves.

4. FUTURE WORK

Our major future work is to design a prior-free auction for bi-dimensional sellers which is truthful and competitive for all concave revenue curves. Subsequently, we would like to design prior-free procurement auctions for the more generic setting where each seller can announce discounts based on the volume of supply.

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