Strategyproof Matching with Minimum Quotas and Initial Endowments*

(Extended Abstract)

Naoto Hamada, Ryoji Kurata, Suguru Ueda, Takamasa Suzuki, and Makoto Yokoo
Kyushu University
Motooka 744, Fukuoka, Japan
{nhamada@agent.,kurata@agent.,ueda@,tsuzuki@,yokoo@}inf.kyushu-u.ac.jp

ABSTRACT

Although minimum quotas are important in many real-world markets, existing strategyproof mechanisms require an unrealistic assumption that all students consider all schools acceptable (and vice-versa). We develop a strategyproof matching mechanism called Priority-List based Deferred Acceptance mechanism with Minimum Quotas (PLDA-MQ), which works under more realistic assumptions: (i) a student considers (at least) one particular school, which we call her initial endowment school, acceptable, and vice-versa, and (ii) the initial endowments satisfy all the minimum quotas. We require a matching to respect initial endowments; each student must be assigned to a school that is at least as good as her initial endowment. PLDA-MQ obtains the student-optimal matching within all matchings that respect minimum quotas/initial endowments and satisfies a stability requirement called Priority-List based (PL-) stability.

CCS Concepts

•**Computing methodologies** → **Multi-agent systems;**
•**Applied computing** → *Economics;*

Keywords

Two-sided matching; Deferred acceptance algorithm; Minimum quotas

1. INTRODUCTION

The theory of matching has been extensively developed for markets in which schools have maximum quotas that cannot be exceeded [4]. However, in many real-world markets, *minimum* quotas are also present [2]. In existing works, to guarantee that mechanisms obtain feasible matchings (which respect minimum quotas), it is required that all students consider all schools *acceptable* and vice-versa. This requirement seems unrealistic in many applications. For example, in a public school choice program, it is not likely that a student is willing to attend a public school located very far away from her residence, unless the school offers some very appealing characteristics.

In this paper, we develop a mechanism that works under much more realistic assumptions. We require that a student considers at least one particular school acceptable, which we call her initial endowment school. We assume each school can consider some students unacceptable. However, we require that each school consider its initial endowment students acceptable. We also assume that the matching corresponds to the initial endowments is feasible, i.e., it satisfies all minimum/maximum quotas.

In general, fairness and standard nonwastefulness, both of which compose stability, are incompatible when minimum quotas are imposed [1]. Thus, in this paper, we introduce alternative definitions called Priority-List based (PL-) fairness and PL-nonwastefulness. They compose PL-stability.

We set our research goal to develop a strategyproof mechanism that improves students' welfare while satisfying the required properties. Our newly developed mechanism obtains the *student-optimal* matching within all matchings that satisfy these properties.

2. MODEL

A matching market is given by $(S, C, X, \omega, \succ_S, \succ_C, \succ_{PL}, q_C, p_C)$. $S = \{s_1, s_2, \ldots, s_n\}$ is a finite set of students. $C = \{c_1, c_2, \ldots, c_m\}$ is a finite set of schools. $X \subseteq S \times C$ is a finite set of contracts. Contract $x = (s, c) \in X$ represents that student s is assigned to school c. For any $X' \subseteq X$, let X'_s denote $\{(s, c) \in X' \mid c \in C\}$. Also, let X'_c denote $\{(s, c) \in X' \mid s \in S\}$. $\omega : S \to C$ is an initial endowment function. $\omega(s)$ returns $c \in C$, which is s's initial endowment. Let X^* denote $\bigcup_{s \in S}\{(s, \omega(s))\}$, i.e., X^* is the set of contracts, where each element is a contract between a student and her initial endowment school.

$\succ_S = (\succ_{s_1}, \succ_{s_2}, \ldots, \succ_{s_n})$ is a profile of the students' preferences. For each student s, $\succ_s$ represents the preference of s over X_s. We assume $\succ_s$ is strict for each s. We say $(s, c) \in X_s$ is acceptable for s if $(s, c) \succ_s (s, \omega(s))$ or $c = \omega(s)$ holds. $\succ_C = (\succ_{c_1}, \succ_{c_2}, \ldots, \succ_{c_m})$ is a profile of the schools' priorities. For each school c, $\succ_c$ represents the priority of c over $X_c \cup \{(\phi, c)\}$, where (ϕ, c) represents an outcome such that c is unmatched. We assume $\succ_c$ is strict for each c. Contract (s, c) is acceptable for c if $(s, c) \succ_c (\phi, c)$ or $\omega(s) = c$ holds. We assume each contract x in X_c is acceptable for c. This is without loss of generality because if some contract is unacceptable to a school, we assume it is not included in X. $\succ_{PL}$ is a serial order over X called *priority list (PL)*,

*This work was partially supported by JSPS KAKENHI Grant Number 24220003 and 15K16058.

Appears in: *Proceedings of the 15th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2016), J. Thangarajah, K. Tuyls, C. Jonker, S. Marsella (eds.), May 9–13, 2016, Singapore.*

which represents a tie-breaking order among contracts. We assume $\succ_{PL}$ respects the initial endowments, i.e., for each $x \in X^*$ and $x' \in X \setminus X^*$, $x \succ_{PL} x'$ holds. Also, we assume $\succ_{PL}$ respects $\succ_C$, i.e., for any $(s,c),(s',c) \in X \setminus X^*$, $(s,c) \succ_{PL} (s',c)$ holds iff $(s,c) \succ_c (s',c)$ holds.

$q_C = (q_{c_1}, q_{c_2}, \ldots, q_{c_m})$ is a vector of the schools' maximum quotas. Also, $p_C = (p_{c_1}, p_{c_2}, \ldots, p_{c_m})$ is a vector of the schools' minimum quotas. We assume $\sum_{c \in C} p_c \leq n \leq \sum_{c \in C} q_c$ holds.

X' is *feasible* if it satisfies the following conditions: (i) for all $s \in S$, $X'_s = \{x\}$ and x is acceptable for s, and (ii) for all $c \in C$, $p_c \leq |X'_c| \leq q_c$ holds. We call a feasible set of contracts a *matching*. We assume X^* is feasible, i.e., for each $c \in C$, $p_c \leq |X^*_c| \leq q_c$ holds.

For set of matchings $\mathcal{X}$, $X' \in \mathcal{X}$ is *student-optimal* within $\mathcal{X}$ if it satisfies the following condition: for all $X'' \in \mathcal{X}$ and for all $s \in S$, where $X'_s = \{x'\}$ and $X''_s = \{x''\}$, either $x' \succ_s x''$ or $x' = x''$. There is a chance that no student-optimal matching exists in $\mathcal{X}$. If a student-optimal matching does exist in $\mathcal{X}$, it must be unique.

A *mechanism* is a function that takes a profile of students' preferences as input and returns a matching. We say a mechanism is *strategyproof* if no student ever has any incentive to misreport her preference, regardless of what the other students report.

We say student s has justified envy toward $s' \neq s$ in matching X', where $(s,c) \in X'$, $(s',c') \in X' \setminus X^*$, and $(s,c') \in X \setminus X'$, if $(s,c') \succ_s (s,c)$ and $(s,c') \succ_{c'} (s',c')$ hold. We say student s has justified envy toward $s' \neq s$ in matching X' based on PL, where $(s,c),(s',c') \in X'$ and $(s,c'') \in X \setminus X'$, if $(s,c'') \succ_s (s,c)$, $|X'_{c''}| < q_{c''}$, $|X'_{c'}| > p_{c'}$, and $(s,c'') \succ_{PL} (s',c')$ hold. Matching X' is *PL-fair* if no student has justified envy or justified envy based on PL.

Student s claims an empty seat of c' in matching X' based on PL, where $(s,c) \in X'$ and $(s,c') \in X \setminus X'$, if $(s,c') \succ_s (s,c)$, $|X'_{c'}| < q_{c'}$, $|X'_c| > p_c$ and $(s,c') \succ_{PL} (s,c)$ hold. Matching X' is *PL-nonwasteful* if no student claims an empty seat based on PL.

We say a matching is *PL-stable* if it is PL-fair and PL-nonwasteful. We say a mechanism is PL-stable if it always gives a PL-stable matching.

3. PRIORITY-LIST BASED DEFERRED ACCEPTANCE MECHANISM WITH MINIMUM QUOTAS (PLDA-MQ)

PLDA-MQ is built upon choice functions $Ch_S : 2^X \to 2^X$ and $Ch_C : 2^X \to 2^X$. Ch_S is a choice function of students S, which is a union of individual choice functions, i.e., $Ch_S(X') := \bigcup_{s \in S} Ch_s(X')$. Let $\widehat{X}'_s$ denote $\{x \in X'_s \mid x$ is acceptable for $s\}$. $Ch_s(X')$ returns $\{(s,c)\}$, where (s,c) is s's most preferred contract in $\widehat{X}'_s$ (or $\emptyset$ if $\widehat{X}'_s$ is $\emptyset$).

Ch_C is a choice function of schools C, which is defined as follows.

Definition 1 (Choice function of schools C). *$Ch_C(X')$ is defined as follows:*

1. *$Y \leftarrow \emptyset$.*
2. *Remove (s,c) from X' such that (s,c) has the highest priority based on PL in X'. If there exists no such contract, terminate the procedure and return Y. Otherwise, $Z \leftarrow Y \cup \{(s,c)\}$.*
3. *If $|Z_c| \leq q_c$ and $\sum_{c' \in C} \max(|Z_{c'}|, p_{c'}) \leq n$, then $Y \leftarrow Z$. Go to 2.*

The PLDA-MQ is one instance of the generalized Deferred Acceptance mechanism presented in [3], which is defined as follows.

Definition 2 (PLDA-MQ).

Step 1 *$Re \leftarrow \emptyset$.*

Step 2 *$X' \leftarrow Ch_S(X \setminus Re)$, $X'' \leftarrow Ch_C(X')$.*

Step 3 *If $X' = X''$, then return X', otherwise, $Re \leftarrow Re \cup (X' \setminus Ch_C(X'))$, go to Step 2.*

Here Re represents the rejected contracts. A student cannot choose a contract in Re. First, students propose a set of contracts X' that are most preferred and not rejected so far. Then schools choose X'', which is a subset of X'. If no contract is rejected, the mechanism terminates. Otherwise, the rejected contracts are added to Re, and the mechanism repeats the same procedure.

From Definition 1, it is clear that PLDA-MQ respects q_C. Also, since $n - \sum_{c' \neq c} |Z_{c'}| \geq p_c$ holds, at least p_c students will be allocated to c. Furthermore, a student is never rejected by her initial endowment school. Thus, the outcome of PLDA-MQ is always feasible.

About PLDA-MQ, the following theorems hold.

Theorem 1. *PLDA-MQ obtains a feasible set of contracts.*

Theorem 2. *PLDA-MQ is strategyproof, PL-stable, and obtains the student-optimal matching within all the PL-stable matchings.*

4. CONCLUSIONS

In this paper, we developed a new strategyproof mechanism called PLDA-MQ that can work under more realistic assumptions than existing mechanisms. PLDA-MQ satisfies required properties (respecting minimum quotas/initial endowments, PL-stability), and obtains the student-optimal matching within all matchings that satisfy the required properties. Our future works include developing a mechanism that respects minimum quotas/initial endowments and is more efficient than PLDA-MQ, although it may not be fair.

REFERENCES

[1] L. Ehlers, I. E. Hafalir, M. B. Yenmez, and M. A. Yildirim. School choice with controlled choice constraints: Hard bounds versus soft bounds. *Journal of Economic Theory*, 153:648–683, 2014.

[2] D. Fragiadakis, A. Iwasaki, P. Troyan, S. Ueda, and M. Yokoo. Strategyproof matching with minimum quotas. *ACM Transactions on Economics and Computation*, 4(1):6:1–6:40, 2015.

[3] J. W. Hatfield and P. R. Milgrom. Matching with contracts. *American Economic Review*, 95(4):913–935, 2005.

[4] A. E. Roth and M. A. O. Sotomayor. *Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis (Econometric Society Monographs)*. Cambridge University Press., 1990.