use this to exclude Pareto-dominated alternatives, and to require f to always pick from the top cycle.

For some purposes it will be useful not to include the mutex clauses in our final formula. Models of this formula then correspond to *set-valued* voting rules that satisfy participation interpreted according to the optimistic preference extension. See Section 7 for results in this setting.

5.2 SAT Solving and MUS Extraction

The formulas we have obtained above are all given in *conjunctive normal form* (CNF), and thus can be evaluated without further transformations by any off-the-shelf SAT solver. In order to physically produce a CNF formula as described, we employ a straightforward Python script that performs a breadth-first search to discover all weighted tournaments with up to n voters (see Algorithm 1 for a schematic overview of the program). The script outputs a CNF formula in the standard DIMACS format, and also outputs a file that, for each variable $x_{T,\mathtt{x}}$, records the tournament T and alternative $\mathtt{x}$ it denotes. This is necessary because the DIMACS format uses uninformative variable descriptors (consecutive integers) and memorizing variable meanings allows us to interpret the output of the SAT solver.

Algorithm 1 Generate formula for up to n voters

```
T_0 ← {weighted tournament on {a, b, c, d} with
        all edges having weight 0}.
for k = 1, ..., n do
    T_k ← ∅
    for T ∈ T_{k−1} do
        for ≽ ∈ R do
            Calculate T' := T + ≽
            Add T' to T_k
            Write non-empty_{T'}, mutex_{T'}, condorcet_{T'}
            Write participation_{T,≽}
```

As an example, the output formula for $n = 15$ in DIMACS format has a size of about 7 GB and uses 50 million variables and 2 billion clauses, taking 6.5 hours to write. Plingeling [3], a popular SAT solver that we used for all results in this paper, solves this formula in 50 minutes of wall clock time, half of which is spent parsing the formula.

In case a given instance is satisfiable, the solver returns a satisfying assignment, giving us an existence proof and a concrete example for a voting rule satisfying participation (and any further requirements imposed). In case a given instance in unsatisfiable, we would like to have short certificate of this fact as well. One possibility for this is having the SAT solver output a resolution proof (in DRUP format, say). This yields a machine-checkable proof, but has two major drawbacks: the generated proofs can be uncomfortably large [24], and they do not give human-readable insights about *why* the formula is unsatisfiable.

We handle this problem by computing a *minimal unsatisfiable subset (MUS)* of the unsatisfiable CNF formula. An MUS is a subset of the clauses of the original formula which itself is unsatisfiable, and is minimally so: removing any clause from it yields a satisfiable formula. We used the tools MUSer2 [2] and MARCO [27] to extract MUSes. If an unsatisfiable formula admits a very small MUS, it is often possible to obtain a human-readable proof of unsatisfiability from it [8, 4].

Note that for purposes of extracting human-readable proofs, it is desirable for the MUS to be as small as possible, and also to refer to as few different tournaments as possible. The first issue can be addressed by running the MUS extractor repeatedly, instructing it to order clauses randomly (note that clause sets of different cardinalities can be minimally unsatisfiable with respect to set inclusion); similarly, we can use MARCO to enumerate all MUSes and look for small ones. The second issue can be addressed by computing a *group MUS*: here, we partition the clauses of the CNF formula into *groups*, and we are looking for a minimal set of groups that together are unsatisfiable. In our case, the clauses referring to a given tournament T form a group. In practice, finding a group MUS first and then finding a standard (clause-level) MUS within the group MUS yielded sets of size much smaller than MUSes returned without the intermediate group-step (often by a factor of 10).

To translate an MUS into a human-readable proof, we created another program that visualized the MUS in a convenient form.[1] Indeed, this program outputs the 'proof diagrams' like Figure 1 that appear throughout this paper (though we re-typeset them). We think that interpreting these diagrams is quite natural (and is perhaps even easier than reading a textual translation). More importantly, the automatically produced graphs allowed us to quickly judge the quality of an extracted MUS.

5.3 Incremental Proof Discovery

The SAT encoding described in Section 5.1 only concerns pairwise voting rules, yet none of the (negative) results in this paper require or use this assumption. This is the product of multiple rounds of generating and evaluating SAT formulas, extracting MUSes, and using the insights generated by this as 'educated guesses' to solve more general problems.

Following the process as described so far led to a proof that for 4 alternatives and 12 voters, there is no pairwise Condorcet extension that satisfies participation. That proof used the assumption of pairwiseness, i.e., it assumed that the voting rule returns the same alternative on profiles inducing the same weighted tournament. However, intriguingly, the preference profiles mentioned in the proof did not contain all $4! = 24$ possible preference relations over $\{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}\}$. In fact, it only used 10 of the possible orders. Further, each profile included $R_0 = \{\mathtt{abdc}, \mathtt{bdca}, \mathtt{cabd}, \mathtt{dcab}\}$ as a subprofile. As we argued at the start of Section 5.1, it is intractable to search over the entire space of preference profiles. On the other hand, it is much easier to merely search over all extensions of R_0 that contain at most $n = 12$ voters and only contain copies of the 10 orders previously identified. The SAT formula produced by doing exactly this turned out to be unsatisfiable, and a small MUS extracted from it gave rise to Theorem 3.

The proof of Theorem 6 for 17 voters was obtained by running Algorithm 1 with T_0 initialized to the weighted tournament induced by the initial profile R used in the proof of Theorem 3. Before finding this tournament, we tried various other tournaments as T_0, including ones featuring in Moulin's original proof, and ones occurring at other steps in the proof of Theorem 3, but R turned out to give the best

[1]Roughly, the visualization program proceeds by drawing an edge for every $\mathsf{participation}_{T,\succcurlyeq}$ clause that occurs in the MUS, and marks the nodes for which $\mathsf{condorcet}_T$ clauses appear in the MUS.

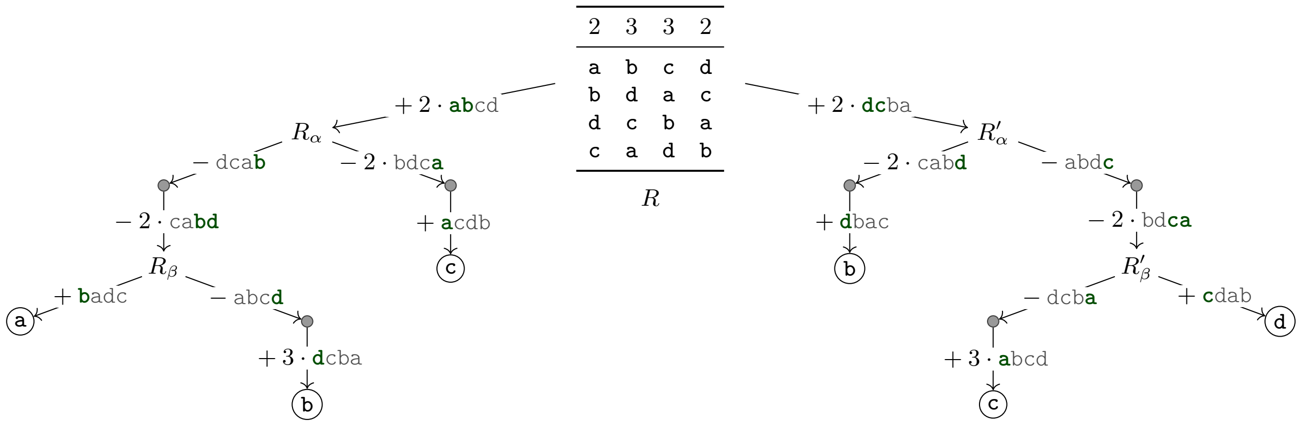


Figure 1: Computer-aided proof of Theorem 3 in graphical form, showing that there is no Condorcet extension that satisfies participation for $m \geqslant 4$ and $n \geqslant 12$. See Section 4 for an explanation of how to read this diagram.

(and indeed a tight) bound, and additionally exhibits a lot of symmetry that was also present in the MUS we extracted.

6. MAIN RESULT

We are now in a position to state and prove our main claim that Condorcet extensions cannot avoid the no-show paradox for 12 or more voters (when there are at least 4 alternatives), and that this result is optimal.

THEOREM 3. *There is no Condorcet extension that satisfies participation for $m \geqslant 4$ and $n \geqslant 12$.*

PROOF. The proof follows the structure depicted in Figure 1. Let R be the preference profile shown there.

Since R remains fixed after relabelling alternatives according to $\texttt{abcd} \mapsto \texttt{dcba}$, we may assume without loss of generality that $f(R) \in \{\texttt{a}, \texttt{b}\}$. (An explicit proof in case $f(R) \in \{\texttt{c}, \texttt{d}\}$ is indicated in Figure 1.)

By participation, it follows from $f(R) \in \{\texttt{a}, \texttt{b}\}$ that also $f(R_\alpha := R + 2 \cdot \texttt{abcd}) \in \{\texttt{a}, \texttt{b}\}$ since the voters with preferences abcd cannot be worse off by joining the electorate. If $f(R_\alpha) = \texttt{a}$, again by participation, removing 2 voters with preferences bdca does not change the winning alternative (so $f(R_\alpha - 2 \cdot \texttt{bdca}) = \texttt{a}$), and neither does adding acdb, so $f(R_\alpha - 2 \cdot \texttt{bdca} + \texttt{acdb}) = \texttt{a}$, which, however, is in conflict with $R_\alpha - 2 \cdot \texttt{bdca} + \texttt{acdb}$ having a Condorcet winner, c.

Thus we must have $f(R_\alpha) = \texttt{b}$, which implies that $f(R_\alpha - \texttt{dcab}) = \texttt{b}$, and thus $f(R_\beta := R_\alpha - \texttt{dcab} - 2 \cdot \texttt{cabd}) \in \{\texttt{b}, \texttt{d}\}$.

We again proceed by cases: If $f(R_\beta) = \texttt{b}$, we can add a voter badc to obtain a profile with Condorcet winner a, which contradicts participation. But then, if $f(R_\beta) = \texttt{d}$, we get that $f(R_\beta - \texttt{abcd}) = \texttt{d}$ and, by another application of participation, that $f(R_\beta - \texttt{abcd} + 3 \cdot \texttt{dcba}) = \texttt{d}$ in contrast to the existence of Condorcet winner b, a contradiction.

If $m > 4$, we add bad alternatives $\texttt{x}_1, \texttt{x}_2, \ldots, \texttt{x}_{m-4}$ to the bottom of R and all other voters. By Lemma 1, f chooses from $\{\texttt{a}, \texttt{b}, \texttt{c}, \texttt{d}\}$ at each step, completing the proof. □

The following result establishes that our bound on the number of voters is tight. A very useful feature of our computer-aided approach is that we can easily add additional requirements for the desired voting rule. Two common requirements for voting rules are that they should only return alternatives that are *Pareto-optimal* and contained in the *top cycle* (also known as the *Smith set*) (see, e.g., [17]).

THEOREM 4. *There is a Condorcet extension f that satisfies participation for $m = 4$ and $n \leqslant 11$. Moreover, f is pairwise, Pareto-optimal, and a refinement of the top cycle.*

The Condorcet extension f is given as a look-up table, which is derived from the output of a SAT solver. The look-up table lists all 1,204,215 weighted tournaments inducible by up to 11 voters and assigns each an output alternative (see Figure 2 for an excerpt). The relevant text file has a size of 28 MB (gzipped 4.5 MB) and is available as part of an arXiv version of this paper [10].

Comparing this voting rule with known voting rules, it turns out that it picks a maximin winner in 99.8% and a Kemeny winner in 98% of all weighted tournaments. Moreover, the rule agrees with the maximin rule with lexicographic tie-breaking on 95% of weighted tournaments. The similarity with the maximin rule is interesting insofar as a well-documented flaw of the maximin rule is that it fails to be a refinement of the top cycle (and may even return Condorcet losers). Our computer-generated rule always picks from the top cycle while it remains very close to the maximin rule.

80% of the considered weighted tournaments admit a Condorcet winner, which uniquely determines the output of the rule; this can be used to reduce the size of the look-up table.

7. SET-VALUED VOTING RULES

A drawback of voting rules, as we defined them so far, is that that the requirement to always return a single winner is in conflict with basic fairness conditions, namely anonymity and neutrality. A large part of the social choice literature therefore deals with set-valued voting rules, where ties are usually assumed to be eventually broken by some tie-breaking mechanism.

A *set-valued voting rule* (sometimes known as a voting *correspondence* or as an *irresolute* voting rule) is a function $F\colon \mathcal{R}^{\mathcal{E}(\mathcal{N})} \to 2^A \setminus \{\emptyset\}$ that assigns each preference profile R a non-empty set of alternatives. The function F is a *(set-valued) Condorcet extension* if for every preference profile R that admits a Condorcet winner $\texttt{x}$, we have $F(R) = \{\texttt{x}\}$.

Following the work of Pérez [29] and Jimeno et al. [22], we seek to study the occurrence of the no-show paradox in

a,#1,(1,1,1,1,1,1)	a,#11,(9,11,3,9,1,-9)
a,#1,(1,1,1,1,1,-1)	a,#11,(11,9,3,7,1,-9)
a,#1,(1,1,1,-1,1,1)	c,#11,(5,-9,-1,-11,-1,7)
a,#1,(1,1,1,-1,-1,1)	c,#11,(5,-9,-1,-11,-1,5)
a,#1,(1,1,1,1,-1,-1)	c,#11,(3,-11,-1,-9,1,7)
a,#1,(1,1,1,-1,-1,-1)	c,#11,(3,-11,-3,-9,1,7)
b,#1,(-1,1,1,1,1,1)	c,#11,(3,-11,-3,-11,-1,7)
b,#1,(-1,1,1,1,1,-1)	b,#11,(-1,3,-5,-3,5,-3)
b,#1,(-1,-1,1,1,1,1)	b,#11,(-3,3,-7,-3,5,-3)
b,#1,(-1,-1,-1,1,1,1)	b,#11,(-3,1,-7,-3,5,-3)
b,#1,(-1,1,-1,1,1,-1)	c,#11,(-3,1,-5,-5,5,-1)
b,#1,(-1,-1,-1,1,1,-1)	a,#11,(3,7,11,-3,9,11)
c,#1,(1,-1,1,-1,1,1)	a,#11,(3,7,11,-3,9,9)
c,#1,(1,-1,1,-1,-1,1)	a,#11,(3,7,11,-5,9,11)

Figure 2: Excerpt of look-up table giving a pairwise Condorcet extension satisfying participation for $n \leqslant 11$ voters (from Theorem 4). Each row lists a weighted tournament as $(g_R(\mathtt{a},\mathtt{b}), g_R(\mathtt{a},\mathtt{c}), g_R(\mathtt{a},\mathtt{d}), g_R(\mathtt{b},\mathtt{c}), g_R(\mathtt{b},\mathtt{d}), g_R(\mathtt{c},\mathtt{d}))$ with a chosen alternative, and with the number of voters inducing the tournament.

this setting. To do so, we need to define appropriate notions of participation, and for this we will need to specify agents' preferences over *sets* of alternatives. Here, we use the *optimistic* and *pessimistic preference extensions*. An optimist prefers sets with better most-preferred alternative, while a pessimist prefers sets with better least-preferred alternative. For example, if $U = \{\mathtt{a},\mathtt{b},\mathtt{d}\}$ and $V = \{\mathtt{b},\mathtt{c}\}$, then an optimist with preferences abcd prefers U to V, while a pessimist prefers V to U. With these notions, we extend the participation property to set-valued voting rules.

Definition 3. *A set-valued voting rule F satisfies* optimistic participation *if* $\max_{\succcurlyeq_i} F(R + \succcurlyeq_i) \succcurlyeq_i \max_{\succcurlyeq_i} F(R)$.

A set-valued voting rule F satisfies pessimistic participation *if* $\min_{\succcurlyeq_i} F(R) \succcurlyeq_i \min_{\succcurlyeq_i} F(R - i)$.

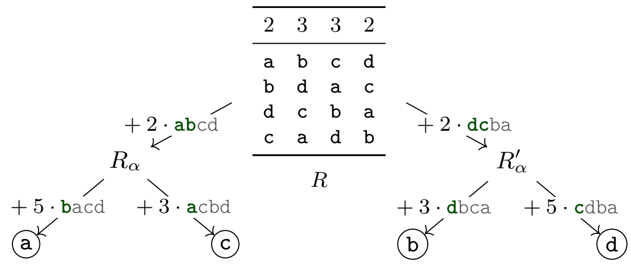
A set-valued voting rule F is called *resolute* if it always selects a single alternative, so that for all R we have $|F(R)| = 1$. A (single-valued) voting rule f is naturally identified with a resolute set-valued voting rule F; if f satisfies participation, then this F satisfies both optimistic and pessimistic participation. Hence, by Theorem 4, there is a (resolute) set-valued Condorcet extension F that satisfies both optimistic and pessimistic participation. However, there might be hope that allowing voting rules to be irresolute also allows for participation to be attainable for more voters, and indeed this is the case.

Theorem 5. *There is a set-valued Condorcet extension F that satisfies optimistic participation for $m = 4$ and $n \leqslant 16$, and also is Pareto-optimal and a refinement of the top cycle.*

The SAT solver indicates that no such set-valued voting rule is pairwise. Theorem 5 is optimal in the sense that optimistic participation cannot be achieved if we allow for one more voter.

Theorem 6. *There is no set-valued Condorcet extension that satisfies optimistic participation for $m \geqslant 4$ and $n \geqslant 17$.*

Proof. Let F be such a function, and consider the following 10-voter profile R:



Suppose that either $\mathtt{a} \in F(R)$ or $\mathtt{b} \in F(R)$. (The case of $\mathtt{c} \in F(R)$ or $\mathtt{d} \in F(R)$ is symmetric.) Then let $R_\alpha := R + 2\cdot\mathtt{abcd}$. By optimistic participation, we then have either $\mathtt{a} \in F(R_\alpha)$ or $\mathtt{b} \in F(R_\alpha)$. If we had $\mathtt{a} \in F(R_\alpha)$, then also $\mathtt{a} \in F(R_\alpha + 3 \cdot \mathtt{acbd})$ but this profile has Condorcet winner c, and if $\mathtt{b} \in F(R_\alpha)$ then also $\mathtt{b} \in F(R_\alpha + 5 \cdot \mathtt{bacd})$ but this profile has Condorcet winner a. This is a contradiction.

This argument extends to more than 4 alternatives by appealing to a set-valued analogue of Lemma 1. □

Inspecting Moulin's original proof [28] shows that it also establishes an impossibility for optimistic participation (for 25 voters). Apparently unaware of this, Jimeno et al. [22] explicitly establish such a result for 27 voters and 5 alternatives. It is worth observing that each step of the proof of Theorem 6 involves *adding* voters to the current profile, and we never remove voters. In light of Definition 3, this is the reason why the proof establishes a result for optimistic participation. If we restrict ourselves to deleting voters, we obtain a result for pessimistic participation.

Theorem 7. *There is no set-valued Condorcet extension that satisfies pessimistic participation for $m \geqslant 4$ and $n \geqslant 14$. On the other hand, for $m = 4$ and $n \leqslant 13$, there exists such a set-valued voting rule.*

Proof Sketch. The proof has a similar structure to the proof of Theorem 3, displayed in Figure 1. The initial profile of this proof is $R + 2 \cdot \mathtt{abcd} + 2 \cdot \mathtt{dcba}$, taking R to be the profile of Figure 1. We further replace proof steps in which voters are added by similar ones where voters are deleted, and invoke pessimistic participation at each such step to obtain a contradiction. □

This result strengthens a result of Jimeno et al. [22], who show that for $m \geqslant 5$ no set-valued Condorcet extension satisfying a property called "weak translation invariance" can also satisfy pessimistic participation. Our proof does not need the extra assumption, already works for $m = 4$ alternatives, and uses just 14 instead of 971 voters.[2]

As previously observed, adding voters in our impossibility proofs corresponds to optimistic participation, while removing voters corresponds to pessimistic participation. In the proof of Theorem 3, we use both operations, which allows for a tighter bound of just 12 voters. In the set-valued setting, we can formulate this result in a slightly stronger way.

[2]The large number of voters is due to several applications of the "weak translation invariance" axiom, each of which adds $5! = 120$ voters to the preference profile under consideration.

THEOREM 8. *There is no set-valued Condorcet extension that satisfies optimistic and pessimistic participation simultaneously for $m \geqslant 4$ and $n \geqslant 12$. On the other hand, for $m = 4$ and $n \leqslant 11$ such a set-valued rule exists (and also is Pareto-optimal and a refinement of the top cycle).*

PROOF. Use the proof of Theorem 3, invoking optimistic participation for edges labelled with the addition of a voter (+), and invoking pessimistic participation for edges labelled with removal of a voter (−). On the other hand, the (single-valued) voting rule of Theorem 4 clearly satisfies both optimistic and pessimistic participation. □

The preference extension combining the optimistic and pessimistic preference extension is also known as the *Egli-Milner extension*.

8. PROBABILISTIC VOTING RULES

A *probabilistic voting rule* (also known as a *social decision scheme*) assigns to each preference profile R a probability distribution (or *lottery*) over A. Thus, a probabilistic voting rule might assign a fair coin flip between $\mathtt{a}$ and $\mathtt{b}$ as the outcome of an election.

Formally, let $\Delta(A) = \{\mathbf{p} : A \to [0,1] : \sum_{\mathtt{x} \in A} \mathbf{p}(\mathtt{x}) = 1\}$ be the set of lotteries over A; a lottery $\mathbf{p} \in \Delta(A)$ assigns probability $\mathbf{p}(\mathtt{x})$ to alternative $\mathtt{x}$. A probabilistic voting rule f is a function $f : \mathcal{R}^{\mathcal{E}(\mathcal{N})} \to \Delta(A)$. In this context, we say that f is a *Condorcet extension* if $f(R)$ puts probability 1 on the Condorcet winner of R whenever it exists: if R admits $\mathtt{x}$ as the Condorcet winner, then $f(R)(\mathtt{x}) = 1$.

As in the set-valued case, we need a notion of comparing outcomes in order to extend the definition of participation. Here, we use the concept of *stochastic dominance (SD)*.

DEFINITION 4. *Let $\succcurlyeq \in \mathcal{R}$ be a preference relation over A, and let $\mathbf{p}, \mathbf{q} \in \Delta(A)$ be lotteries. Then $\mathbf{p}$ is (weakly) SD-preferred over $\mathbf{q}$ by $\succcurlyeq$ if for each alternative $\mathtt{x}$, we have*

$$\textstyle\sum_{\mathtt{y} \succcurlyeq \mathtt{x}} \mathbf{p}(y) \geqslant \sum_{\mathtt{y} \succcurlyeq \mathtt{x}} \mathbf{q}(y).$$

In this case, we write $\mathbf{p} \succcurlyeq^{\mathrm{SD}} \mathbf{q}$.

For example, the lottery $\frac{2}{3}\mathtt{a} + \frac{1}{3}\mathtt{c}$ is SD-preferred to the lottery $\frac{1}{3}\mathtt{a} + \frac{1}{3}\mathtt{b} + \frac{1}{3}\mathtt{c}$ by a voter with preferences $\mathtt{abcd}$. A voter with preferences $\mathtt{bacd}$ will feel the other way around. The main appeal of stochastic dominance stems from the following equivalence: $\mathbf{p} \succcurlyeq^{\mathrm{SD}} \mathbf{q}$ if and only if $\mathbf{p}$ yields at least as much von-Neumann-Morgenstern utility as $\mathbf{q}$ under any utility function that is consistent with the ordinal preferences $\succcurlyeq$. Using this notion of comparing lotteries, we can define participation analogously to the previous settings.

DEFINITION 5. *A probabilistic voting rule f satisfies* strong SD-participation *if $f(R) \succcurlyeq_i^{\mathrm{SD}} f(R-i)$ for all $R \in \mathcal{R}^N$ and $i \in N$ with $N \in \mathcal{E}(\mathcal{N})$.*

Any (single-valued) voting rule f can be seen as a probabilistic voting rule that puts probability 1 on its chosen alternative. If f satisfies participation, then this derived probabilistic voting rule is easily seen to satisfy strong SD-participation. Hence Theorem 4 gives us a probabilistic Condorcet extension that satisfies strong SD-participation for $n \leqslant 11$ voters and $m = 4$ alternatives.

We now establish a connection between strong SD-participation and the set-valued notions of participation that we considered in Section 7. This connection will allow us to translate the impossibility results we obtained there to the probabilistic setting. To set up this connection, let us define the *support* of a lottery $\mathbf{p} \in \Delta(A)$ to be $\mathrm{supp}(\mathbf{p}) := \{\mathtt{x} \in A : \mathbf{p}(\mathtt{x}) > 0\}$.

PROPOSITION 1. *Let f be a probabilistic voting rule satisfying strong SD-participation. Let $F = \mathrm{supp} \circ f$ be the support of f, i.e., $F(R) = \mathrm{supp}(f(R))$ for all profiles R. Then F satisfies both optimistic and pessimistic participation.*

PROOF. We only verify optimistic participation; the pessimistic case is similar. Let R be a preference profile with electorate $N \in \mathcal{E}(\mathcal{N})$, and let $i \in \mathcal{N} \setminus N$ be a voter with preferences $\succcurlyeq_i$. Let $\mathtt{x} = \max_{\succcurlyeq_i} F(R)$, and let $U = \{\mathtt{y} : \mathtt{y} \succcurlyeq_i \mathtt{x}\}$. We need to show that $\max_{\succcurlyeq_i} F(R + \succcurlyeq_i) \succcurlyeq_i \mathtt{x}$, by finding an alternative $\mathtt{y} \in U$ that is in the support of $f(R + \succcurlyeq_i)$.

But since f satisfies strong SD-participation, we have

$$\textstyle\sum_{\mathtt{y} \in U} f(R + \succcurlyeq_i)(\mathtt{y}) \geqslant \sum_{\mathtt{y} \in U} f(R)(\mathtt{y}) > 0,$$

where the strict inequality follows from the definition of the support and of $\mathtt{x}$. Hence some alternative from U is in the support of $f(R + \succcurlyeq_i)$, as required. □

Putting these results together with the impossibility result of Theorem 8, we obtain the following.

THEOREM 9. *There is no probabilistic Condorcet extension that satisfies strong SD-participation for $n \geqslant 12$ and $m \geqslant 4$. On the other hand, for $m = 4$ and $n \leqslant 11$, such a probabilistic voting rule exists.*

Theorem 9 resolves an open problem mentioned by Brandl et al. [5, Sec. 6].

9. CONCLUSIONS AND FUTURE WORK

We have given tight results delineating in which situations no-show paradoxes must occur. As such, our results nicely complement recent advances to satisfy Condorcet-consistency and participation by exploiting uncertainties of the voters about their preferences or about the voting rule's tie-breaking mechanism [4, 5, 6].

Due to unmanageable branching factors when there are 5 alternatives (and hence $5! = 120$ possible preference relations), we were unable to check using our approach whether no-show paradoxes occur with even less voters when the number of alternatives grows. It would be interesting to gain a deeper understanding of the computer-generated Condorcet extension that satisfies participation for up to 11 voters. So far, we only know that it (slightly) differs from all Condorcet extensions that are usually considered in the literature. As a first step, it would be desirable to obtain a representation of this rule that is more concise than a look-up table.

Another interesting topic for future research is to find optimal bounds for a variant of the no-show paradox due to Sanver and Zwicker [32], in which participation is weakened to half-way monotonicity. Their proof requires 702 voters.

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