Resource Logics with a Diminishing Resource

Extended Abstract

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ABSTRACT

Model-checking resource logics with production and consumption of resources is a computationally hard and often undecidable problem. We show that it is more feasible under the assumption that there is at least one *diminishing resource*, that is, a resource which is consumed by every action.

KEYWORDS

Model-checking; resources

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1 INTRODUCTION

There has been a considerable amount of work on resource logics interpreted over structures where agents' actions produce and consume resources, for example [2, 3, 6–9, 12–14, 17–19]. There exists also a large body of related work on reachability and non-termination problems in energy games and games on vector addition systems with state [1, 11, 15, 16, 21]. The resource logics considered in this paper are extensions of the Alternating Time Temporal Logic (ATL), [10]. For ATL under imperfect information and with perfect recall uniform strategies, ATL_{iR}, the model-checking problem is undecidable for three or more agents [20]. It is however decidable in the case of bounded strategies [23].

In this paper we introduce a special kind of models for resource logics satisfying a restriction that one of the resources is always consumed by each action. This is a very natural setting that occurs in many verification problems. One obvious example of such a resource is time. Other examples include systems where agents have a non-rechargeable battery and where all actions consume energy, e.g., nodes in a wireless sensor network; and systems where agents have a store of propellant that cannot be replenished during the course of a mission and all actions of interest involve manoeuvring, e.g., a constellation of satellites. We call this special resource that is consumed by all actions a *diminishing resource*.

We study $\text{RB} \pm \text{ATL}^\#$ and $\text{RB} \pm \text{ATL}^\#_{iR}$, diminishing resource versions of Resource-Bounded Alternating Time Temporal Logic (RB±ATL) [5]. The model-checking problem for RB±ATL is known to be 2EXPTIME-complete [6], while $\text{RB} \pm \text{ATL}^\#$ model-checking is in PSPACE if resource bounds are written in unary. In the case of $\text{RB} \pm \text{ATL}^\#_{iR}$, the result of [23] does not apply immediately because the bound is not fixed in advance, but its model checking problem is decidable in EXPSPACE given encoding in unary. We also study $\text{RAL}^\#$, a diminishing resource version of Resource Agent Logic (RAL) [13]. Decidability of $\text{RAL}^\#$ follows from the result on the decidability of RAL on bounded models [13], but the PSPACE upper bound (for unary encoding) is new.

2 $\text{RB} \pm \text{ATL}^\#$

The syntax of $\text{RB} \pm \text{ATL}^\#$ is defined relative to the following sets: $Agt = \{a_1, \ldots, a_n\}$ is a set of n agents, $Res = \{res_1, \ldots, res_r\}$ is a set of r resource types, Π is a set of propositions, and $\mathcal{B} = \mathbb{N}^{Res^{Agt}}$ is a set of resource bounds (resource allocations to agents). Elements of $\mathcal{B}$ are vectors of length n where each element is a vector of length r. We will denote by $\mathcal{B}_A$ (for $A \subseteq Agt$) the set of possible resource allocations to agents in A. Formulas of $\text{RB} \pm \text{ATL}^\#$ are defined by:

$$\phi, \psi ::= p \mid \neg\phi \mid \phi \vee \psi \mid \langle\langle A^b \rangle\rangle \bigcirc \phi \mid \langle\langle A^b \rangle\rangle \phi \,\mathcal{U}\, \psi \mid \langle\langle A^b \rangle\rangle \phi \,\mathcal{R}\, \psi$$

where $p \in \Pi$, $A \subseteq Agt$, and $b \in \mathcal{B}_A$. $\langle\langle A^b \rangle\rangle \bigcirc \phi$ means that a coalition A can ensure that the next state satisfies ϕ under resource bound b. $\langle\langle A^b \rangle\rangle \phi \,\mathcal{U}\, \psi$ means that A has a strategy to enforce ψ while maintaining the truth of ϕ, and the cost of this strategy is at most b. $\langle\langle A^b \rangle\rangle \phi \,\mathcal{R}\, \psi$ means that A has a strategy to maintain ψ until and including the time when ϕ becomes true, or to maintain ψ forever if ϕ never becomes true, and the cost of this strategy is at most b. The language is interpreted on the following structures:

Definition 2.1. A resource-bounded concurrent game structure with diminishing resource ($\text{RB-CGS}^\#$) is a tuple $M = (Agt, Res, S, \Pi, \pi, Act, d, c, \delta)$ where:

- Agt, Res and Π are as above; the first resource type in Res is the distinguished diminishing resource;
- S is a non-empty finite set of states;
- $\pi : \Pi \to \wp(S)$ is a truth assignment that associates each $p \in \Pi$ with a subset of states where it is true;
- Act is a non-empty set of actions;
- $d : S \times Agt \to \wp(Act) \setminus \{\emptyset\}$ is a function that assigns to each $s \in S$ a non-empty set of actions available to each agent $a \in Agt$.
- $c : S \times Act \to \mathbb{Z}^r$ is a partial function that maps a state s and an action σ to a vector of integers, where a positive (negative) integer in position i indicates consumption (production) of resource r_i by the action. The first position in the vector is always at most -1.
- $\delta : S \times Act^{|Agt|} \to S$ is a partial function that maps every $s \in S$ and $\sigma \in d(s, a_1) \times \cdots \times d(s, a_n)$ to a state resulting from executing σ in s.

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In what follows, we use the usual point-wise notation for vector comparison and addition, and, given a function f returning a vector, we denote by f_i the function that returns the i-th component of the vector returned by f. Given an RB-CGS$^\#$ M and a state $s \in S$, a *joint action by a coalition* $A \subseteq Agt$ is a tuple $\sigma = (\sigma_a)_{a \in A}$ such that $\sigma_a \in d(s, a)$. The set of all joint actions for A at state s is denoted by $D_A(s)$. Given a joint action by Agt, $\sigma \in D_{Agt}(s)$, σ_A denotes the joint action executed by A as part of σ: $\sigma_A = (\sigma_a)_{a \in A}$. The set of all possible outcomes of a joint action $\sigma \in D_A(s)$ at state s is: $out(s, \sigma) = \{s' \in S \mid \exists \sigma' \in D_{Agt}(s) : \sigma = \sigma'_A \wedge s' = \delta(s, \sigma')\}$. A *strategy for a coalition* $A \subseteq Agt$ in an RB-CGS$^\#$ M is a mapping $F_A : S^+ \to Act^{|A|}$ such that, for every $\lambda \in S^+$, $F_A(\lambda) \in D_A(\lambda[|\lambda|])$. A computation λ is consistent with a strategy F_A iff, for all i, $1 \leq i < |\lambda|$, $\lambda[i+1] \in out(\lambda[i], F_A(\lambda[1, i]))$. We denote by $out(s, F_A)$ the set of all computations λ starting from s that are consistent with F_A. Given a bound $b \in \mathcal{B}$, a computation $\lambda \in out(s, F_A)$ is b-consistent with F_A iff, for every $i \geq 0$, for every $a \in A$, $b_a - \sum_{j=0}^{j=i-1} c(F_a(\lambda[0, j])) \geq c(F_a(\lambda[0, i]))$.

A computation λ is b-maximal for a strategy F_A if it cannot be extended further while remaining b-consistent. The set of all maximal computations starting from state s that are b-consistent with F_A is denoted by $out(s, F_A, b)$.

Given an RB-CGS$^\#$ M and a state s of M, the truth of an RB±ATL$^\#$ formula ϕ with respect to M and s is defined as follows (omitting the cases for propositions, $\neg$ and $\wedge$):

- $M, s \models \langle\langle A^b \rangle\rangle \bigcirc \phi$ iff $\exists$ strategy F_A such that for all b-maximal $\lambda \in out(s, F_A, b)$: $|\lambda| \geq 2$ and $M, \lambda[2] \models \phi$;
- $M, s \models \langle\langle A^b \rangle\rangle \phi\, \mathcal{U}\, \psi$ iff $\exists$ strategy F_A such that for all b-maximal $\lambda \in out(s, F_A, b)$, $\exists i$ such that $1 \leq i \leq |\lambda|$: $M, \lambda[i] \models \psi$ and $M, \lambda[j] \models \phi$ for all $j \in \{1, \ldots, i-1\}$.
- $M, s \models \langle\langle A^b \rangle\rangle \phi\, \mathcal{R}\, \psi$ iff $\exists$ strategy F_A such that for all b-maximal $\lambda \in out(s, F_A, b)$, either $\exists i$ such that $1 \leq i \leq |\lambda|$: $M, \lambda[i] \models \phi$ and $M, \lambda[j] \models \psi$ for all $j \in \{1, \ldots, i\}$; or, $M, \lambda[j] \models \psi$ for all j such that $1 \leq j \leq |\lambda|$.

The following theorem is proved by demonstrating a model-checking algorithm for RB ± ATL$^\#$, see [4]:

Theorem 2.2. *The model-checking problem for RB ± ATL$^\#$ is decidable in PSPACE (under unary encoding).*

3 RB ± ATL$^\#_{iR}$

In this section, we study RB ± ATL$^\#_{iR}$, RB ± ATL$^\#$ with imperfect information and perfect recall. To model imperfect information, RB-CGS$^\#$ are extended with an indistinguishability relation $\sim_a$ on states, for every agent a. This relation can be lifted to finite sequences of states. Strategies under imperfect information should be *uniform*: if agent a is uncertain whether the history so far is λ or λ' ($\lambda \sim_a \lambda'$), then the strategy for a should return the same action for both λ and λ': $F_a(\lambda) = F_a(\lambda')$. A strategy F_A for a group of agents A is uniform if it is uniform for every agent in A. In what follows, we consider *strongly uniform* strategies [22], that require the existence of a uniform strategy from all indistinguishable states:

- $M, s \models \langle\langle A^b \rangle\rangle \bigcirc \phi$ under strong uniformity iff there exists a uniform strategy, F_A, such that, for all $s' \sim_a s$ where $a \in A$, for all $\lambda \in out(s', F_A, b)$, $|\lambda| > 1$ and $M, \lambda[2] \models \phi$.

The truth definitions for $\langle\langle A^b \rangle\rangle \phi\, \mathcal{U}\, \psi$ and $\langle\langle A^b \rangle\rangle \phi\, \mathcal{R}\, \psi$ are also modified to require the existence of a *uniform* strategy from all states s' indistinguishable from s by any $a \in A$.

Theorem 3.1. *The model-checking problem for RB ± ATL$^\#_{iR}$ is decidable in EXPSPACE (under unary encoding).*

4 RAL$^\#$

RAL$^\#$ is obtained by modifying the definition of RAL [13] for the diminishing resource setting. The sets Agt, Res, and Π are as before. An *endowment (function)* $\eta : Agt \times Res \to \mathbb{N}$ assigns resources to agents: $\eta_a(r) = \eta(a, r)$ is the amount of resource agent a has of resource type r. En denotes the set of all possible endowments. Formulas of RAL$^\#$ are defined by:

$$\phi, \psi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid \langle\langle A \rangle\rangle_B^{\downarrow} \bigcirc \phi \mid \langle\langle A \rangle\rangle_B^{\eta} \bigcirc \phi \mid \langle\langle A \rangle\rangle_B^{\downarrow} \phi \mathcal{U} \psi \mid \langle\langle A \rangle\rangle_B^{\eta} \phi \mathcal{U} \psi \mid \langle\langle A \rangle\rangle_B^{\downarrow} \phi \mathcal{R} \psi \mid \langle\langle A \rangle\rangle_B^{\eta} \phi \mathcal{R} \psi$$

where $p \in \Pi$, $A, B \subseteq Agt$, and $\eta \in$ En. Unlike in RB ± ATL$^\#$, in RAL$^\#$ there are two types of cooperation modalities, $\langle\langle A \rangle\rangle_B^{\downarrow}$ and $\langle\langle A \rangle\rangle_B^{\eta}$. In both cases, the actions performed by agents in $A \cup B$ consume and produce resources (actions by agents in $Agt \setminus (A \cup B)$ do not change their resource endowment). The meaning of $\langle\langle A \rangle\rangle_B^{\eta} \varphi$ is otherwise the same as in RB ± ATL$^\#$. The formula $\langle\langle A \rangle\rangle_B^{\downarrow} \varphi$ requires that the strategy uses the resources *currently* available to the agents.

The models of RAL$^\#$ are RB-CGS$^\#$. Strategies are also defined as for RB ± ATL$^\#$. However, to evaluate formulas with a down arrow, such as $\langle\langle A \rangle\rangle_B^{\downarrow} \bigcirc \varphi$, we need the notion of *resource-extended computations.* A *resource-extended* computation $\lambda \in (S \times \text{En})^+$ is a sequence over $S \times$ En such that the restriction to states (the first component), denoted by $\lambda|_S$, is a path in the underlying model. The projection of λ to the second component is denoted by $\lambda|_{\text{En}}$. A (η, s_A, B)*-computation*, λ, is a resource-extended computation iff for all $i = 1, \ldots$ with $\lambda[i] := (s_i, \eta^i)$ there is an action profile $\sigma \in d(\lambda|_S[i])$ such that:

- $\eta^0 = \eta$ (η describes the initial resource distribution);
- $F_A(\lambda|_S[1, i]) = \sigma_A$ (A follow their strategy);
- $\lambda|_S[i+1] = \delta(\lambda|_S[i], \sigma)$ (transition according to σ);
- for all $a \in A \cup B$: $\eta_a^i \geq c(\lambda|_S[i], \sigma_a)$ (each agent has enough resources to perform its action);
- for all $a \in A \cup B$: $\eta_a^{i+1} = \eta_a^i - c(\lambda|_S[i], \sigma_a)$ (resources are updated);
- for all $a \in Agt \setminus (A \cup B)$ and $r \in Res$: $\eta_a^{i+1}(r) = \eta_a^i(r)$ (the resources of agents not in $A \cup B$ do not change).

$out(s, \eta, F_A, B)$ is the set of all (η, F_A, B)-computations starting in s. The truth definition is given with respect to a model, a state, and an endowment η:

- $M, s, \eta \models \langle\langle A \rangle\rangle_B^{\downarrow} \bigcirc \varphi$ iff there is a strategy F_A for A such that for all $\lambda \in out(s, \eta, F_A, B)$, $|\lambda| > 1$ and $M, \lambda|_S[2], \lambda|_{\text{En}}[2] \models \varphi$

and similarly for $\langle\langle A \rangle\rangle_B^{\downarrow} \varphi \mathcal{U} \psi$ and $\langle\langle A \rangle\rangle_B^{\downarrow} \varphi \mathcal{R} \psi$. The cases for $\langle\langle A \rangle\rangle_B^{\zeta} \bigcirc \varphi$, $\langle\langle A \rangle\rangle_B^{\zeta} \varphi \mathcal{U} \psi$, $\langle\langle A \rangle\rangle_B^{\zeta} \varphi \mathcal{R} \psi$ quantify over $\lambda \in out(s, \zeta, F_A, B)$.

Theorem 4.1. *The model-checking problem for RAL$^\#$ is decidable in PSPACE (under unary encoding).*

REFERENCES

[1] Parosh Aziz Abdulla, Richard Mayr, Arnaud Sangnier, and Jeremy Sproston. 2013. Solving Parity Games on Integer Vectors. In *CONCUR'13*, Vol. 8052. Springer, 106–120.

[2] Natasha Alechina, Nils Bulling, Stéphane Demri, and Brian Logan. 2016. On the Complexity of Resource-Bounded Logics. In *Proceedings of the 10th International Workshop on Reachability Problems (RP 2016) (Lecture Notes in Computer Science)*, Kim Guldstrand Larsen, Igor Potapov, and Jirí Srba (Eds.), Vol. 9899. Springer, 36–50.

[3] Natasha Alechina, Nils Bulling, Brian Logan, and Hoang Nga Nguyen. 2017. The virtues of idleness: A decidable fragment of resource agent logic. *Artificial Intelligence* 245 (2017), 56–85. https://doi.org/10.1016/j.artint.2016.12.005

[4] Natasha Alechina and Brian Logan. 2018. Resource Logics with a Diminishing Resource. *CoRR* (2018). To appear.

[5] Natasha Alechina, Brian Logan, Hoang Nga Nguyen, and Franco Raimondi. 2014. Decidable Model-Checking for a Resource Logic with Production of Resources. In *Proceedings of the 21st European Conference on Artificial Intelligence (ECAI-2014)*, Torsten Schaub, Gerhard Friedrich, and Barry O'Sullivan (Eds.). IOS Press, 9–14.

[6] Natasha Alechina, Brian Logan, Hoang Nga Nguyen, and Franco Raimondi. 2017. Model-checking for Resource-Bounded ATL with production and consumption of resources. *J. Comput. System Sci.* 88 (September 2017), 126–144. https://doi.org/doi.org/10.1016/j.jcss.2017.03.008

[7] Natasha Alechina, Brian Logan, Hoang Nga Nguyen, and Abdur Rakib. 2009. A Logic for Coalitions with Bounded Resources. In *Proceedings of the Twenty First International Joint Conference on Artificial Intelligence (IJCAI 2009)*, Craig Boutilier (Ed.), Vol. 2. AAAI Press, 659–664.

[8] Natasha Alechina, Brian Logan, Hoang Nga Nguyen, and Abdur Rakib. 2010. Resource-bounded alternating-time temporal logic. In *Proceedings of the Ninth International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2010)*, Wiebe van der Hoek, Gal Kaminka, Yves Lespérance, Michael Luck, and Sandip Sen (Eds.). IFAAMAS, 481–488.

[9] Natasha Alechina, Brian Logan, Hoang Nga Nguyen, and Abdur Rakib. 2011. Logic for coalitions with bounded resources. *Journal of Logic and Computation* 21, 6 (2011), 907–937.

[10] Rajeev Alur, Thomas A. Henzinger, and Orna Kupferman. 2002. Alternating-Time Temporal Logic. *Journal of the ACM* 49, 5 (2002), 672–713.

[11] Tomas Brázdil, Petr Jancar, and Antonin Kucera. 2010. Reachability Games on Extended Vector Addition Systems with States. In *ICALP'10*, Vol. 6199. Springer, 478–489.

[12] Nils Bulling and Berndt Farwer. 2010. Expressing Properties of Resource-Bounded Systems: The Logics RBTL and RBTL*. In *Computational Logic in Multi-Agent Systems - 10th International Workshop, CLIMA X, Revised Selected and Invited Papers (Lecture Notes in Computer Science)*, Vol. 6214. 22–45.

[13] Nils Bulling and Berndt Farwer. 2010. On the (Un-)Decidability of Model Checking Resource-Bounded Agents. In *Proceedings of the 19th European Conference on Artificial Intelligence (ECAI 2010)*, Helder Coelho, Rudi Studer, and Michael Wooldridge (Eds.). IOS Press, 567–572.

[14] Nils Bulling and Valentin Goranko. 2013. How to Be Both Rich and Happy: Combining Quantitative and Qualitative Strategic Reasoning about Multi-Player Games (Extended Abstract). In *Proceedings of the 1st International Workshop on Strategic Reasoning (SR 2013) (Electronic Proceedings in Theoretical Computer Science)*, Fabio Mogavero, Aniello Murano, and Moshe Y. Vardi (Eds.), Vol. 112. 33–41.

[15] Jean-Baptiste Courtois and Sylvain Schmitz. 2014. Alternating Vector Addition Systems with States. In *MFCS'14*, Vol. 8634. Springer, 220–231.

[16] Aldric Degorre, Laurent Doyen, Raffaella Gentilini, Jean-François Raskin, and Szymon Toruńczyk. 2010. Energy and Mean-Payoff Games with Imperfect Information. In *Computer Science Logic, 24th International Workshop, CSL 2010, 19th Annual Conference of the EACSL, Brno, Czech Republic, August 23-27, 2010. Proceedings (Lecture Notes in Computer Science)*, Anuj Dawar and Helmut Veith (Eds.), Vol. 6247. Springer, 260–274. https://doi.org/10.1007/978-3-642-15205-4

[17] Dario Della Monica and Giacomo Lenzi. 2012. On a Priced Resource-bounded Alternating μ-Calculus. In *Proceedings of the 4th International Conference on Agents and Artificial Intelligence (ICAART 2012)*, Joaquim Filipe and Ana L. N. Fred (Eds.). SciTePress, 222–227.

[18] Dario Della Monica, Margherita Napoli, and Mimmo Parente. 2011. On a Logic for Coalitional Games with Priced-Resource Agents. *Electronic Notes in Theoretical Computer Science* 278 (2011), 215–228.

[19] Dario Della Monica, Margherita Napoli, and Mimmo Parente. 2013. Model checking coalitional games in shortage resource scenarios. In *Proceedings of the Fourth International Symposium on Games, Automata, Logics and Formal Verification (GandALF 2013) (Electronic Proceedings in Theoretical Computer Science)*, Gabriele Puppis and Tiziano Villa (Eds.), Vol. 119. 240–255.

[20] Catalin Dima and Ferucio Laurentiu Tiplea. 2011. Model-checking ATL under Imperfect Information and Perfect Recall Semantics is Undecidable. *CoRR* abs/1102.4225 (2011). http://arxiv.org/abs/1102.4225

[21] Marcin Jurdzinski, Ranko Lazić, and Sylvain Schmitz. 2015. Fixed-Dimensional Energy Games are in Pseudo-Polynomial Time. In *ICALP'15*, Vol. 9135. Springer, 260–272.

[22] Bastien Maubert and Sophie Pinchinat. 2014. A General Notion of Uniform Strategies. *IGTR* 16, 1 (2014). https://doi.org/10.1142/S0219198914400040

[23] Steen Vester. 2013. Alternating-time temporal logic with finite-memory strategies. In *Proceedings Fourth International Symposium on Games, Automata, Logics and Formal Verification, GandALF 2013, Borca di Cadore, Dolomites, Italy, 29-31th August 2013 (EPTCS)*, Gabriele Puppis and Tiziano Villa (Eds.), Vol. 119. 194–207. https://doi.org/10.4204/EPTCS.119