On the Impact of Buyers Preselection in Pricing Problems

Extended Abstract

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ABSTRACT

We investigate the problem of preselecting a subset of buyers participating in a market so as to optimize the performance of stable outcomes. We consider four scenarios arising from the combination of two stability notions, item and bundle envy-freeness, with the two classical objective functions, i.e., the social welfare and the seller's revenue. When adopting the notion of item envy-freeness, we prove that, for both the two objective functions, the problem cannot be approximated within $n^{1-\varepsilon}$ for any $\varepsilon > 0$, and provide tight or nearly tight approximation algorithms. We also prove that maximizing the seller's revenue is NP-hard even for a single buyer, thus closing a longstanding open question. Under bundle envy-freeness, instead, we show how to transform in polynomial time any stable outcome for a market involving only a subset of buyers to a stable one for the whole market without worsening its performance, both for the social welfare and the seller's revenue. This transformation implies that, although in this case buyer preselection cannot improve the performance, it can still be used as an algorithmic tool for computing good stable outcomes when preselection is not allowed. In fact, it can be first exploited to simplify the combinatorics of the problem, and then for mapping back the computed solution to one encompassing all the buyers. Finally, we consider multi-unit markets, where all items are of the same type and are assigned the same price. For this specific case, we show that buyer preselection can improve the performance of stable outcomes in all of the four considered scenarios, and design corresponding approximation algorithms.

KEYWORDS

Pricing Problems; Envy-freeness; Revenue Maximization; Social Welfare Maximization

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1 INTRODUCTION

Determining an efficient pricing strategy is a fundamental problem in many business activities, as it affects both the seller's revenue and the customers' or buyers' satisfaction. Usually, optimal prices are the result of a challenging counterbalancing process: selecting low prices, for instance, may be profitable for the seller when it attracts considerably more customers, but, at the same time, in case of limited supply, it may leave some buyers unsatisfied, thus generating discontent. In particular, this happens when a customer is negated the right to buy her preferred set of items, or even any item at all, despite the fact that she is willing to pay for the posted prices. In this case she is often called *loser*, as opposed to a customer receiving items, called *winner*. For such a reason, pricing problems are traditionally considered under the hypothesis of ***envy-freeness*** [20, 26], which prescribes that, once a pricing strategy has been established, items have to be allocated to buyers in such a way that no one would prefer a different set of items.

However, if from the one hand safeguarding the losers' interests shelters the seller from possible future losses due to their dissatisfaction, on the other hand, a result by [5] shows that, in certain markets, an intrinsic and unavoidable hurdle to the construction of a good quality envy-free solution may come from the presence of a set of "disturbing" customers, that is, a set of buyers such that at least one of them gets envious in any assignment of sufficiently high revenue. This observation naturally leads to the following intriguing question: *"What happens if envy-freeness is restricted to apply only to the set of winners? Can the seller raise enough more revenue (with certainty) today to compensate the (uncertain) future loss of potential customers?"*. Such a relaxed form of envy-freeness models indeed the situation in which the seller is allowed the freedom to discard any subset of buyers from the given instance, so as to get rid of envious losers in the assignment she would like to propose.

1.1 Our Contribution

Motivated by the above discussion, we introduce and investigate the *buyer preselection problem* in which, given a pricing problem P with n buyers and m items, we are interested in computing the best possible envy-free solution that can be achieved by removing any arbitrary subset of buyers from P. We consider four scenarios arising from the combination of two stability notions, called item and bundle envy-freeness, respectively, with the two classical objective functions, namely, the social welfare and the seller's revenue.

In an item envy-free allocation, given a pricing of the items, each buyer gets the subset maximizing her utility among all possible subsets that can be created from the set of available items; in a bundle envy-free allocation, no buyer gets a better utility by receiving the bundle allocated to any winner. Observe that these allocations are always guaranteed to exist, as it suffices to assign all items an arbitrarily high price, so that no winner is possible.

For item envy-free allocations and both objective functions, we show that the buyer preselection problem cannot be approximated within $n^{1-\epsilon}$ for every $\epsilon > 0$, unless P = ZPP. On the positive side, under the objective of social welfare, we design an n-approximation algorithm, while, for the case of revenue maximization, we give an $O(n \log m)$-approximation. In particular, these results are obtained as follows: all but one buyer are discarded from the given instance, so that we are left with a pricing problem with a single buyer. While such a problem is solvable in polynomial time under the objective of social welfare, for revenue maximization, it already exhibits challenging combinatorial structures and, to the best of our knowledge, has been considered before only in [3]. In this paper an $O(\log m)$-approximation is provided, but no lower bounds on the problem complexity are given. We show that the problem is NP-hard, thus solving the corresponding open problem raised by the authors.

We stress that efficient preselection can be profitable under two orthogonal directions: from the one hand, the removal of a subset of pathological envious buyers may increase the value of the optimal solution; from the other hand, even when this does not happen or it has only a modest impact, simplifying the combinatorial structure defined by the valuation functions of the winners may lead to the design of better approximation algorithms. In fact, for the two preselection problems obtained by considering bundle envy-free allocations, we show how to transform in polynomial time any allocation which is bundle envy-free only for the subset of the winners to a bundle envy-free allocation for all buyers without worsening its performance. Hence, although this transformation implies that, in this case, buyer preselection cannot improve the performance of stable outcomes, it can be used to map any bundle envy-free allocation for a subset of winners obtained through preselection back to a bundle envy-free allocation involving all buyers.

Finally, we consider the multi-unit case, where all items are of the same type and are assigned the same price. We show how preselection can improve the revenue and the social welfare of both item and bundle envy-free solutions. In particular, for item envy-free allocations, we show a tight multiplicative factor of m for both objective functions. For bundle envy-free allocations, we show a lower multiplicative bound of 2 for both the revenue and the social welfare, and prove that it is tight for the objective of revenue maximization. We also provide tight results on the complexity of computing optimal solutions for the buyer preselection problem under envy-freeness.

1.2 Related Work

The literature on envy-free pricing is so vast that it cannot be exhaustively covered here. For such a reason, we simply refer to the achievements which are mostly related to the model of [23] we consider in this paper.

For the social welfare maximization, the VCG mechanism [15, 21, 27] provides an optimal solution to the envy-free pricing problem. However, while this mechanism is efficiently computable in markets with unit-demand buyers, yet for single-minded ones its computation becomes NP-hard. Approximate solutions are still possible in this case thanks to the results of [2]. Also Walrasian Equilibria [28] provide an optimal solution to the problem [4]; however, they are guaranteed to exist only under very stringent hypothesis on the buyers' valuation functions [22].

For the revenue maximization, [3, 7, 14, 23, 24] design logarithmic approximation algorithms for various special cases of the problem. Relative hardness results have been given by [6, 8–10, 16]. Further variants have been considered by [1, 5, 11–13, 17].

[18] propose an interesting relaxation of the notion of Walrasian Equilibrium, called Combinatorial Walrasian Equilibrium (CWE), obtained by grouping items into bundles so as to induce a "reduced market" to which, then, applying the notion of Walrasian Equilibrium. They show the existence of a CWE yielding a 2-approximation of the optimal social welfare and that of a CWE yielding a logarithmic approximation of the optimal revenue.

Finally, [25] study the case of revenue maximization in markets with multi-unit items under both item and bundle envy-freeness when allowing both item and bundle pricing. Such setting has been extended in [19] where the authors consider a social graph of the buyers and envies can arise only between neighbors.

2 CONCLUSIONS AND FUTURE WORK

Many results holding for the item envy-free outcomes and social welfare objective function extend to the notion of Walrasian equilibria, that are item envy-free outcomes with the additional requirement that the market clears, i.e., every unsold item is assigned price zero. In particular, the inapproximability result showing that the buyer preselection problem cannot be approximated within $n^{1-\epsilon}$ for every $\epsilon > 0$, unless P = ZPP, and the n-approximation algorithm for the buyer preselection problem directly extend to Walrasian equilibria. Notice also that for the remaining uncovered cases, that is when the goal is that of optimizing the seller's revenue, there is no reason for requiring market clearance, a condition clearly limiting the power of setting prices so as to maximize the revenue.

The main left open problems are: for the pricing problem defined on markets with a unique buyer, closing the gap between the **NP**-hardness and the logarithmic approximation for the case of revenue maximization and item envy-free solutions; for the multi-unit case with bundle envy-free outcomes, determining an upper bound to the social welfare improvement achievable by preselection and setting the complexity of computing optimal solutions, for both the revenue and the social welfare cases.

REFERENCES

[1] E. Anshelevich, K. Kar, and S. Sekar. 2015. Envy-Free Pricing in Large Markets: Approximating Revenue and Welfare. In *Proceedings of the 42nd International Colloquium on Automata, Languages, and Programming (ICALP 2015)*. Springer, 52–64.

[2] A. Archer, C. H. Papadimitriou, K. Talwar, and É. Tardos. 2003. An Approximate Truthful Mechanism for Combinatorial Auctions with Single Parameter Agents. *Internet Mathematics* 1, 2 (2003), 129–150.

[3] M. F. Balcan, A. Blum, and Y. Mansour. 2008. Item pricing for revenue maximization. In *Proceedings of the 9th ACM Conference on Electronic Commerce (EC)*. 50–59.

[4] S. Bikhchandani and J. W. Mamer. 1997. Competitive equilibrium in an exchange economy with indivisibilities. *Journal of Economic Theory* 74, 2 (1997), 386–413.

[5] V. Bilò, M. Flammini, and G. Monaco. 2017. Approximating the revenue maximization problem with sharp demands. *Theoretical Computer Science* 662 (2017), 9–30.

[6] P. Briest. 2008. Uniform Budgets and the Envy-Free Pricing Problem. In *Proceedings of the 35th International Colloquium on Automata, Languages and Programming (ICALP 2008)*. Springer, 808–819.

[7] P. Briest and P. Krysta. 2006. Single-minded unlimited supply pricing on sparse instances. In *Proceedings of the 17th Annual ACM-SIAM Symposium on Discrete Algorithm (SODA 2006)*. ACM Press, 1093–1102.

[8] P. Chalermsook, J. Chuzhoy, S. Kannan, and S. Khanna. 2012. Improved Hardness Results for Profit Maximization Pricing Problems with Unlimited Supply. In *Proceedings of the 15th International Workshop on Approximation Algorithms for Combinatorial Optimization Problems (APPROX 2012)*. Springer, 73–84.

[9] P. Chalermsook, B. Laekhanukit, and D. Nanongkai. 2013. Graph Products Revisited: Tight Approximation Hardness of Induced Matching, Poset Dimension and More. In *Proceedings of the 24th Annual ACM-SIAM Symposium on Discrete Algorithms (SODA 2013)*. ACM Press, 1557–1576.

[10] P. Chalermsook, B. Laekhanukit, and D. Nanongkai. 2013. Independent Set, Induced Matching, and Pricing: Connections and Tight (Subexponential Time) Approximation Hardnesses. In *Proceedings of the 54th Annual S-ymposium on Foundations of Computer Science (FOCS 2013)*. IEEE Computer Society, 370–379.

[11] N. Chen and X. Deng. 2010. Envy-Free Pricing in Multi-item Markets. In *Proceedings of the 37th International Colloquium on Automata, Languages and Programming (ICALP 2010)*. Springer, 418–429.

[12] N. Chen, X. Deng, P. W. Goldberg, and J. Zhang. 2016. On revenue maximization with sharp multi-unit demands. *Journal of Combinatorial Optimization* 31, 3 (2016), 1174–1205.

[13] N. Chen, A. Ghosh, and S. Vassilvitskii. 2011. Optimal Envy-Free Pricing with Metric Substitutability. *SIAM J. Comput.* 40, 3 (2011), 623–645.

[14] M. Cheung and C. Swamy. 2008. Approximation algorithms for single-minded envy-free profit-maximization problems with limited supply. In *Proceedings of the 49th Annual Symposium on Foundations of Computer Science (FOCS 2008)*. IEEE Computer Society, 35–44.

[15] E. H. Clarke. 1971. Multipart pricing of public goods. *Public Choice* 11 (1971), 17–33.

[16] E. D. Demaine, U. Feige, M. Hajiaghayi, and M. R. Salavatipour. 2008. Combination Can Be Hard: Approximability of the Unique Coverage Problem. *SIAM J. Comput.* 38, 4 (2008), 1464–1483.

[17] M. Feldman, A. Fiat, S. Leonardi, and P. Sankowski. 2012. Revenue maximizing envy-free multi-unit auctions with budgets. In *Proceedings of the 13th ACM Conference on Electronic Commerce, (EC 2012)*. ACM Press, 532–549.

[18] M. Feldman, N. Gravin, and B. Lucier. 2016. Combinatorial Walrasian Equilibrium. *SIAM J. Comput.* 45, 1 (2016), 29–48.

[19] M. Flammini, M. Mauro, and M. Tonelli. To appear. On Social Envy-Freeness in Multi-Unit Markets. In *Proceedings of the Thirty-Second AAAI Conference on Artificial Intelligence (AAAI 2018)*.

[20] D. Foley. 1967. Resource allocation and the public sector. *Yale Economic Essays* 7 (1967), 45–98.

[21] T. Groves. 1973. Incentives in teams. *Econometrica* 41 (1973), 617–631.

[22] F. Gul and E. Stacchetti. 1999. Walrasian equilibrium with gross substitutes. *Journal of Economic Theory* 87 (1999), 95–124.

[23] V. Guruswami, J. D. Hartline, A. R. Karlin, D. Kempe, C. Kenyon, and F. McSherry. 2005. On profit-maximizing envy-free pricing. In *Proceedings of the 16th annual ACM-SIAM symposium on discrete algorithms (SODA 2005)*. ACM Press, 1164–1173.

[24] J. Hartline and Q. Yan. 2011. Envy, truth, and profit. In *Proceedings of the 12th ACM Conference on Electronic Commerce (EC 2011)*. ACM Press, 243–252.

[25] G. Monaco, P. Sankowski, and Q. Zhang. 2015. Revenue Maximization Envy-Free Pricing for Homogeneous Resources. In *Proceedings of the 24th International Joint Conference on Artificial Intelligence (IJCAI 2015)*. 90–96.

[26] H. R. Varian. 1974. Equity, envy, and efficiency. *Journal of Economic Theory* 9 (1974), 63–91.

[27] W. Vickrey. 1961. Counterspeculation, auctions, and competitive sealed tenders. *Journal of Finance* 16 (1961), 8–37.

[28] L. Walras. 1954. *Elements of Pure Economics*. Allen and Unwin.