# Intolerance does not Necessarily Lead to Segregation: A Computer-aided Analysis of the Schelling Segregation Model

Extended Abstract

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## KEYWORDS

Schelling Segregation Model; Convergence; Emergent Properties; Computer-aided Analysis

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## 1 THE SCHELLING SEGREGATION MODEL

Let $N = \{1, 2, \ldots, n\}$ be a set of individuals. We assume these individuals to be arranged on an array - i.e., a permutation of $N$ - and partitioned into $k \leq n$ types. For simplicity, and unless otherwise specified, our study will involve two types of equal size, the $*$'s and the $\circ$'s. An instance of the model could for instance be:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| $*$ | $\circ$ | $\circ$ | $*$ | $*$ | $\circ$ | $\circ$ | $*$ |

We assume each individual to have a **neighbourhood range** $r \in \mathbb{N} \setminus 0$, which is made by those individuals that live at most $r$ positions to the right or $r$ positions to the left.

Individuals have preferences over their neighbours. In particular they have a **preference ratio** $p \in [0, 1]$, i.e., the least proportion of individuals in their neighbourhood that need to share their characteristics, in our case their same type.

In Schelling's model, unhappy individuals are allowed to move to the closest neighbourhood that would make them happy. Specifically, individuals are moved according to the following protocol, which we refer to as the **Schelling turn function**:

(1) Record the set of currently unhappy individuals;
(2) Select the leftmost individual in the set, who is still unhappy in the current configuration and hasn't been selected yet;
(3) Move him or her to the closest neighbourhood that would make the individual happy, jumping over all individuals in between.[1] If he or she cannot be made happy, then he or she does not move;
(4) Repeat from step 2 until all individuals in the set have had the opportunity to move or are happy;
(5) Repeat from step 1 with the new set of unhappy individuals;
(6) Stop if there are no unhappy individuals, or for all of them they cannot move anywhere that would make them happy.

We must now precisely define what a **terminal state** is. According to Schelling, a terminal state is a state that fulfils the following condition: "for all unhappy agents, there is no place they can move that would make them happy". Clearly, the condition is trivially satisfied at states where all agents are happy.

Schelling's claim is that if the individuals all have a preference ratio $p \geq \frac{1}{3}$, any such model will eventually converge into at least a significantly *more* segregated terminal state if not a *totally* segregated state, such as the terminal state above [1, page 159].

Starting with these observations, it's interesting to understand whether segregation, is de facto unavoidable, as Schelling claims, and, if so, whether the segregation rating of a terminal state can be at all limited, and to what extent, by exploring different paths to less segregated terminal states than Schelling's turn function would produce. As such, we aimed to create a simulator to test which terminal states could be achieved using both Schelling's turn function and any other possible move.

## 2 COMPUTER-GENERATED FINDINGS

With the simulators implemented, a large number of simulations were undertaken with random initial state configurations and randomised parameters including population size from between 10 and 50, neighbourhood range between 1 and 10 and the preference ratio of agents being between 0 and 1.0.

Each simulation ran these initial states both to find the Schelling terminal state, i.e., the unique terminal state reached via the Schelling turn function, as well as using the randomised explorer to attempt to find 1,000 alternate paths to potentially different terminal states.

These found terminal states along with relevant data on them were then uploaded to a central database, enabling multiple machines to run large numbers of different simulations at once, whilst only requiring a central database be queried to review the results. In total 130,018 simulations were able to be run and have their results reported on.

The first main discovery was the incredibly small set of unique terminal states that initial states tend to have, with 58.6% of all initial states ran only having a single terminal state.

The second main finding is that whilst increasing preference rating does suddenly result in convergence to highly segregated

[1]Though Schelling is somewhat ambiguous over which direction an agent should move in a case of a tie for closest desired position, the way the examples are resolved suggest that ties are broken going to the right [1].

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| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\bar{1}$ * | $\bar{2}$ * | 3 ○ | 4 ○ | 5 ○ | 6 ○ | 7 ○ | $\bar{8}$ * | $\bar{9}$ * | $\bar{10}$ * |
| $\bar{1}$ * | $\bar{2}$ * | 3 ○ | 4 ○ | 5 ○ | 6 ○ | 8 * | 7 ○ | $\bar{9}$ * | $\bar{10}$ * |
| $\bar{1}$ * | 2 * | 3 ○ | 4 ○ | 5 ○ | 6 ○ | 9 * | $\bar{8}$ * | 7 ○ | $\bar{10}$ * |
| $\bar{1}$ * | 2 * | 3 ○ | 4 ○ | 5 ○ | 8 * | 6 ○ | $\bar{9}$ * | 7 ○ | $\bar{10}$ * |
| $\bar{2}$ * | 3 ○ | 4 ○ | 1 * | 5 ○ | 8 * | 6 ○ | $\bar{9}$ * | 7 ○ | 10 * |
| 3 ○ | 2 * | 4 ○ | 1 * | 5 ○ | 8 * | 6 ○ | 9 * | 7 ○ | 10 * |

**Table 1: An example of an initial state that achieves a totally integrated terminal state from an initially high segregation index (0.78).** $r = 6, p = 0.4$ **and moves resulting in repeated states are forbidden.**

| State | r | p | Initial **s** | Repeats |
|---|---|---|---|---|
| ○○**○**○○* | 6 | 0.40 | 0.44 | ✗ |
| ○*○*○*○***○○ | 4 | 0.36 | 0.27 | ✗ |
| ○**○*○*○○* | 4 | 0.33 | 0.22 | ✓ |
| **○○*○*○*○ | 4 | 0.40 | 0.22 | ✗ |
| *○○*○*○**○ | 6 | 0.36 | 0.22 | ✓ |
| ○○**○*○*○*○* | 2 | 0.27 | 0.18 | ✗ |
| ○**○*○*○*○ | 2 | 0.20 | 0.11 | ✓ |
| *○○*○*○*○* | 4 | 0.40 | 0.11 | ✓ |
| ○*○**○*○*○ | 6 | 0.40 | 0.11 | ✓ |
| *○○*○*○*○* | 2 | 0.20 | 0.11 | ✓ |
| *○*○○*○*○* | 6 | 0.40 | 0.11 | ✓ |
| ○**○*○*○*○* | 8 | 0.38 | 0.10 | ✓ |
| *○*○*○*○**○ | 2 | 0.14 | 0.10 | ✓ |

**Table 2: Sample of initial states that achieved a 0% segregation rating through Schelling's turn function, and if they were also found with repeated states allowed.**

terminal states past a preference ratio of 0.4, once preference ratio moves beyond 0.6, less segregated terminal states can start to reappear.

What is far more surprising and interesting is the very rare existence of states that seemed to have a negative change in segregation from initial state to terminal state by the application of the Schelling terminal function.

The third and possibly most striking finding was that 4,122 (3.17%) states actually found terminal states with a *reduced* segregation rating from the initial state by allowing individuals to move based upon these intolerance preferences.

Indeed, some states even reduced all the way to 0 segregation, that is they reached a fully integrated terminal state. 64 of these states were found through the random explorer, and 14 through Schelling's turn function. Presented in Table 1 is such a case; a fully integrated state is achieved despite a high initial segregation rating, and it terminates with all agents content and a preference ratio above the critical threshold.

Table 2 presents a sample of some of the initial states where totally integrated terminal states were discovered through Schelling's turn function.

Table 3 shows some of the states that can achieve a totally integrated terminal state found by the random path sampler.

It is notable that many of these initial states are in fact only slight variations of each other, sometimes with only a single individual reallocated. It is likely these slight configurations exist on each others path to the terminal state.

There are two other points worth mentioning, both of which can also be noticed on these two tables.

(1) In all cases where segregation reduced preference was never above 0.5.
(2) In all cases where a completely integrated terminal state was found from a initial state that was not, neighbourhood range was always an even number.

The vast amount of explorations carried out suggest that reductions in segregation come together with the two properties above.

| State | r | p | Initial **s** |
|---|---|---|---|
| **○○○○○*** | 6 | 0.40 | 77.78 |
| **○○○○***○○* | 4 | 0.34 | 63.64 |
| **○○*○○***○○○***○○○* | 4 | 0.40 | 57.89 |
| **○*○○○○** | 6 | 0.40 | 55.56 |
| ○*○****○○○ | 6 | 0.40 | 55.56 |
| ○**○**○○○○*○** | 8 | 0.43 | 46.15 |
| ○**○○○**○**○ | 4 | 0.40 | 45.45 |
| *○*○○***○○ | 4 | 0.40 | 44.44 |
| **○○*○**○○ | 2 | 0.26 | 44.44 |
| ○○***○**○○*○**○ | 8 | 0.41 | 42.86 |
| ○**○○○***○○*○*○**○○* | 4 | 0.40 | 42.11 |
| ○**○**○○○○*○*○** | 8 | 0.44 | 40.00 |
| ○○**○*○○**○○**○* | 2 | 0.27 | 40.00 |
| ○*○**○○**○ | 2 | 0.31 | 33.33 |
| *○**○○*○○* | 2 | 0.33 | 33.33 |
| *○○*○*○○** | 2 | 0.30 | 33.33 |
| ○**○○○**○*○*○*○*○○** | 4 | 0.40 | 31.58 |
| *○○○***○*○*○○*○**○*○ | 4 | 0.40 | 31.58 |
| *○*○○**○**○○*○ | 2 | 0.29 | 30.77 |
| ○**○*○*○○**○ | 6 | 0.38 | 27.27 |
| *○*○○○**○*○* | 6 | 0.41 | 27.27 |

**Table 3: Sample of initial states that achieved a 0% segregation rating through randomised exploration.**

However a decisive formal argument would need to be provided in order to confirm this conjecture.

All in all, whilst the presence of states that reduce in segregation is certainly rare, the fact they exist, and that some achieve total integration, is certainly a surprising result given Schelling's claims.

## REFERENCES

[1] Thomas C. Schelling. 1971. Dynamic Models of Segregation. *Journal of Mathematical Sociology* 1 (1971), 143–186.