2.1 Aggregation Rules

The *conjunction rules* return all the models in common among the individual goals, and a chosen default model if there are none.

$$Conj_v(\Gamma) = \begin{cases} \text{Mod}(\gamma_1 \wedge \cdots \wedge \gamma_n) & \text{if non-empty} \\ \{v\} \text{ for } v \in \{0,1\}^m & \text{otherwise} \end{cases}$$

The *threshold rules* are comparable to quota rules in judgment aggregation [7]. Let $\mu_\varphi : \text{Mod}(\varphi) \to \mathbb{R}$ be a function associating to each model v of φ some weight $\mu_\varphi(v)$. Then:

$$TrSh^\mu(\Gamma)_j = \{1\} \text{ iff } \Big(\sum_{i \in \mathcal{N}} (w_i \cdot \sum_{v \in \text{Mod}(\gamma_i)} v(j) \cdot \mu_{\gamma_i}(v))\Big) \geq q_j$$

such that $0 \leq q_j \leq n+1$ for all $j \in \mathcal{I}$ is the quota of issue j, where for each $v \in \text{Mod}(\gamma_i)$ we have $\mu_{\gamma_i}(v) \neq 0$ and $w_i \in [0,1]$ is the individual weight of agent i. For readability, we omit vector $(q_1, \ldots, q_m)$, with the quotas for the issues, from the name of $TrSh^\mu$.

We call *EQuota* rule the $TrSh^\mu$ rules having $\mu_{\gamma_i}(v) = \frac{1}{|\text{Mod}(\gamma_i)|}$ and $w_i = 1$ for all $v \in \text{Mod}(\gamma_i)$ and $i \in \mathcal{N}$. Since $\sum_{v \in \text{Mod}(\gamma_i)} \mu_{\gamma_i}(v) = 1$ for all $i \in \mathcal{N}$, the number of models of a goal formula is irrelevant and agents have an equal impact on the outcome.

Finally, we introduce three alternative definitions of the majority rule. The first one is the *EMaj* (resolute) rule, which is an *EQuota* rule having $q_j = \lceil \frac{n}{2} \rceil$ for all $j \in \mathcal{I}$. The second version of majority is irresolute and it compares for each issue the number of acceptances with the number of rejections, weighting each model of an individual goal in the same way as in *EQuota* rules:

$$TrueMaj(\Gamma) = \Pi_{j \in \mathcal{I}} M(\Gamma)_j$$

where for all $j \in \mathcal{I}$

$$M(\Gamma)_j = \begin{cases} \{x\} & \text{if } \sum_{i \in \mathcal{N}} \frac{m^x_{ij}}{|\text{Mod}(\gamma_i)|} > \sum_{i \in \mathcal{N}} \frac{m^{1-x}_{ij}}{|\text{Mod}(\gamma_i)|} \\ \{0,1\} & \text{otherwise} \end{cases}$$

Thus, for each issue $j \in \mathcal{I}$ we compute the value of $M(\Gamma)_j$ by setting it to $\{1\}$ (respectively, $\{0\}$) if strictly more than half of the agents accept (respectively, reject) issue j, and to $\{0,1\}$ if exactly half of the agents accept/reject. The third version of majority is *2sMaj*, defined as *Maj*(*Maj*(γ_1), ..., *Maj*(γ_n)), where the strict majority rule is applied first to the models of each individual goal and then again to the result obtained in the first step.

3 AXIOMATIC CHARACTERIZATION

The *axiomatic method* in social choice theory evaluates aggregation rules by first defining some general properties (axioms) and then proving whether or not aggregation rules satisfy them. We define adaptations of known axioms for rules aggregating goals.

- (A) An *anonymous* aggregation rule F is such that for any profile Γ and any permutation $\sigma : \mathcal{N} \to \mathcal{N}$, we have that $F(\gamma_1, \ldots, \gamma_n) = F(\gamma_{\sigma(1)}, \ldots, \gamma_{\sigma(n)})$.
- (I) An *independent* aggregation rule F is such that for any two profiles Γ and Γ', for all $j \in \mathcal{I}$ and for all $i \in \mathcal{N}$, we have that $v_i(j) = v'_i(j)$ implies $F(\Gamma)_j = F(\Gamma')_j$.
- (N) A *neutral* aggregation rule F is such that for all profiles Γ, for all two issues $j, k \in \mathcal{I}$, and for all agents $i \in \mathcal{N}$ we have that $v_i(j) = v_i(k)$ implies $F(\Gamma)_j = F(\Gamma)_k$.
- (U) A *unanimous* aggregation rule F is such that for all profiles Γ and for all $j \in \mathcal{I}$, if $m^x_{ij} = 0$ for all $i \in \mathcal{N}$ then $F(\Gamma)_j = \{1-x\}$.
- (PR) Profiles Γ and Γ' are *comparable* if and only if for all $i \in \mathcal{N}$ we have that $|\text{Mod}(\gamma_i)| = |\text{Mod}(\gamma'_i)|$. An aggregation rule satisfies *positive responsiveness* if for all comparable profiles Γ and $\Gamma' = (\gamma_1, \ldots, \gamma'_i, \ldots, \gamma_n)$, for all issues $j \in \mathcal{I}$ and for all $i \in \mathcal{N}$, if $m'^x_{ij} \geq m^x_{ij}$ for $x \in \{0,1\}$, then $[F(\Gamma)_j = \{x\}$ or $F(\Gamma)_j = \{0,1\}]$ implies $F(\Gamma')_j = \{x\}$.
- (E) An aggregation rule F is *egalitarian* if and only if for all profiles Γ, if we construct a profile Γ' with $|\mathcal{N}'| = \text{lcm}(|\text{Mod}(\gamma_1)|, \ldots, |\text{Mod}(\gamma_n)|)$, and for all $i \in \mathcal{N}$ and each $v \in \text{Mod}(\gamma_i)$ we have $\frac{|\mathcal{N}'|}{|\mathcal{N}| \cdot |\text{Mod}(\gamma_i)|}$ agents in $\mathcal{N}'$ voting v in Γ', then $F(\Gamma) = F(\Gamma')$.
- (D) An aggregation rule satisfies *duality* when for all profiles Γ and for all issues $j \in \mathcal{I}$, if $F(\Gamma)_j = \{x\}$ then $F(\overline{\Gamma})_j = \{1-x\}$, where $\overline{\Gamma}$ is such that $\overline{v_i(j)} = (\overline{m^1_{ij}}, \overline{m^0_{ij}}) = (m^0_{ij}, m^1_{ij})$ for all $j \in \mathcal{I}$ and $i \in \mathcal{N}$.

Following the seminal result of May [19], where an axiomatization of the majority rule in the context of voting over two alternatives is provided, we also axiomatically characterize *TrueMaj*, the most intuitive among our proposed generalizations of majority.

THEOREM 3.1. *For arbitrary $\mathcal{N}$ and $\mathcal{I}$, a goal-aggregation rule satisfies* (E), (I), (A), (N), (PR), (U) *and* (D) *if and only if it is TrueMaj.*

4 COMPUTATIONAL COMPLEXITY

The *winner determination* problem asks how hard it is to compute the outcome of aggregation rules [2, 6, 11, 17]. For resolute rules we define it as follows (and analogously for irresolute rules):

WINDET(F)
Input: profile Γ, issue j
Question: Is it the case that $F(\Gamma)_j = \{1\}$?

For a special case of $TrSh^\mu$ we get completeness for the class NP.

THEOREM 4.1. WINDET*($TrSh^\mu$) is* NP*-complete, for $\mu_{\gamma_i}(v) = 1$ constant and $w_i = 1$ for all $i \in \mathcal{N}$.*

Let PP be the complexity class Probabilistic Polynomial Time. Let *TrueMaj** be a resolute version of *TrueMaj* that in case of equal support for issue j returns 0 in the outcome.

THEOREM 4.2. *Problems* WINDET*(EMaj),* WINDET*(TrueMaj*) and* WINDET*(2sMaj) are* PP*-hard.*

Since Γ contains formulas, if a rule has to manipulate the models to compute the outcome some form of satisfiability is needed — thus starting from the complexity class NP. Asking agents to provide the models of their goals would lower the complexity of some results, but the input would become demanding for the agents and also of exponential size in the number of issues in the worst case.

5 CONCLUSIONS

We presented a framework to handle the aggregation of individual goals in a multi-agent setting. We defined a number of procedures taking as input the consistent goal formulas of the agents and returning a collective decision in the form of a set of valuations for the issues at stake. We introduced three alternative definitions of the majority rule and characterized one of them axiomatically. We also studied computationally the problem of determining the outcome for some of our proposed rules.

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