

Figure 1: Prediction error for DSA- $k$  for various values of  $k$

in the protocol's information processing. The key difference lies in how often the agents take cost function changes into consideration. This is controlled by a parameter value we call  $k$ . To evaluate the proposed model, we ran an empirical test using the DSA- $k$  protocol where we varied the value for  $k$  from 2 through 5. These tests were conducted using  $n = 100$  variables,  $|D| = 3$ ,  $p_1 = 0.035$ ,  $\langle c_i = 5 \rangle$  and a *rate* that varied from 0 to 99. For each setting of  $k$  and *rate*, we conducted 30 experiments, each of 1000 steps, where the cost functions were altered, but not added or removed. The results of these tests can be seen in Figure 1. The results clearly show the impact that stagnant information has on the protocol. As the value of  $k$  is increased, a noticeable increase in the error can be seen.

With these results in mind, we revisited the equation for the change in energy caused by work. This equation has two principle components, the convergence rate,  $A$  and the convergence point,  $B$ . By definition, the convergence point is unaffected by delays in information because it represents the solution that the protocol will eventually obtain assuming infinite time. However, it should be clear that delays in processing updates would cause changes to the speed at which a protocol could reach that solution.

Using this reasoning, we speculated that the convergence rate is changed by a factor  $k$ . To include this finding, we altered the change work equation by multiply the convergence rate by the value  $k$ , which has the impact of slowing convergence. This can be seen in the equation  $\frac{dE_W}{dt} = \frac{B-E}{kA}$ . Then by combining the equations we reach a new equation to create a new equilibrium point equation that factors in the impact of information stagnancy. This is seen in the equation  $E_0 = \frac{kAD + \frac{CB}{rate}}{kA + \frac{C}{rate}}$ . In fact, the error for all of the values of  $k$  is nearly constant indicating that the error that does remain is probably caused by the order of the simulation cycle.

## 4 CONCLUSIONS

In this study we showed that the primary cause of prediction error for the equilibrium point of a protocol operating on a DynDCOP is information stagnancy. This error is created by the method used to characterize the converge rate of a protocol. However, this error

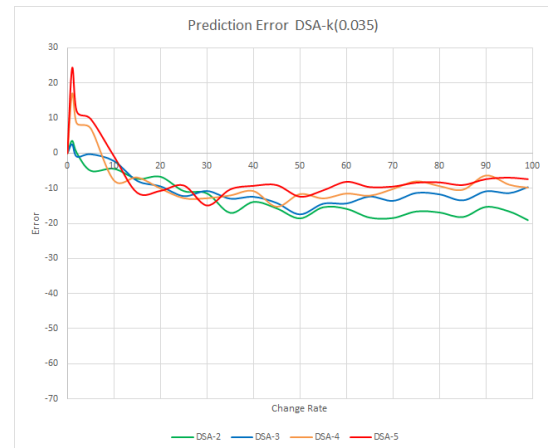


Figure 2: Comparison of predicted versus actual equilibrium points for DSA- $k$  taking into account information stagnancy

can be corrected for by altering the equilibrium prediction equation. In addition, the error can be used to measure the impact that information delay has on the performance of a protocol.

## ACKNOWLEDGMENTS

This work is supported by the National Science Foundation under Grant No.:IIS-1350671

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