Balanced Outcomes in Wage Bargaining

Extended Abstract

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ABSTRACT

Balanced outcomes are a subset of core outcomes that take into consideration fairness and agents' power in bargaining networks. In this paper, following the seminal works by [3] and [6] on modeling and computing balanced outcomes in unit-capacity trading networks, we explore this concept further by considering its generalization in the so-called wage bargaining network where agents on one side (the employers side) may have multiple capacity. It turns out that previous definitions do not trivially extend to this setting. Our first contribution is to incorporate insights from the bargaining theory and define a generalized notion of balanced outcomes in wage bargaining networks.

We then consider computational aspects of this newly proposed solutions. We show that there are polynomial-time combinatorial algorithms to compute such solutions in both unweighted and weighted graphs. Our algorithms and proofs are enabled by novel generalizations of techniques proposed by Kleinberg and Tardos and an original technique proposed in this paper called "loose chain".

KEYWORDS

Bargaining and negotiation; Cooperative games: theory & analysis; Cooperative games: computation

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1 INTRODUCTION

Wage has been one of the primary incentive instruments in social employment relationships. Employers concern their costs and employees pursue higher wages. In this paper, we aim toward a theory to formally investigate the following problem,

what are the stable and fair wage outcomes in a society?

from a perspective that incorporates both game and graph theories. We formally model and justify the concept of minimum wage that each employee should be paid given its ability and position in the social graph and develop efficient algorithms to compute it.

The social employment relationship considered in this paper can be formally modeled as a weighted bipartite graph, with employers on the one side and employees on the other side. If an employee is eligible to work for an employer, there is an edge between them. In addition, the corresponding weight of an edge denotes the value produced by the employee under this matching. In this paper, we assume the values produced by employees are additive, in the sense that the overall value produced by a set of employees is the sum of their individual values. We also assume each employer cannot hire more than a certain amount of employees, which is defined in the graph as its vertex capacity. On the other hand, each employee can only work for at most one employer. For example, in Figure 1, x_1, x_2, x_3, x_4 are four employers with capacity 2, 2, 1, 1 respectively, and $y_1, y_2, y_3, y_4, y_5, y_6$ are six employees with only one capacity each. In addition, the weights of all edges between X and Y are 1.

Given such a social graph instance, we model the wage bargaining interactions between employers and employees as a cooperative game. In the standard cooperative game theory, the most important solution concept is the *core*, which subscribes outcomes in which no subset of agents want to deviate from the current outcome. Even though core outcomes are stable (employees may not switch to other employers), they are still insufficient for the wage bargaining game because they are not necessarily fair in the sense that one side of the graph may get more than they deserve (employees may request for more wages). Over the years, there have been other alternative solution concepts for many different purposes. As we will review later, few of them are satisfactory to model the situation of wage bargaining.

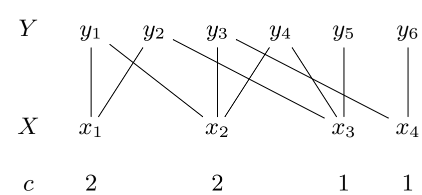

Figure 1: An example of unweighted wage bargaining model

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In this paper, we generalize the notion of *balanced outcome*, originally proposed by Cook and Yamagishi [3] and reinvestigated by Kleinberg and Tardos [6] in the context of social exchange networks, to the context of wage bargaining. In the original model of social exchange networks, each agent has capacity of at most one and the social relation is modeled as a general graph; while in our model, we have a bipartite graph and each employer has multiple capacity.

Although the two models appear to be similar, there is a fundamental difference between them. The size of a coalition in previous models is always restricted to be either one or two, while the size of a coalition in our model (which contains one employee and several employers) can be any positive integers. For the previous case of bargaining between two agents, one can directly apply the standard Nash bargaining solution. In particular, two agents will agree on a division that is the middle point between the extremes of their alternate options. However, in the multi-agent case, alternate options of an agent may intersect with the others'. It is therefore not clear how to divide payoffs regarding to the extremes of alternate options for multiple agents. A careful definition of a multi-agent bargaining solution in this context is in need.

To address this difficulty, our first effort is to define a balanced bargaining solution for general cooperative games. Our definition of *balanced outcome* essentially differs from the existing concepts such as Shapley value [11], bargaining set [1] and nucleolus [10] proposed in the literature.

When defining the new concept, we consider the following desiderata:

(1) It must be a non-trivial subset of the core;
(2) It must reduce to the definition of balanced outcomes in social exchange network;
(3) It is efficiently computable in wage bargaining network.

It can be verified that none of the concepts above satisfy all the desiderata.

Our goal is then to solve the structure of all balanced outcomes and efficiently find the optimal balanced outcome for either the employees or the employers. Starting from the special wage bargaining problem characterized by unweighted graphs with unique perfect matching, we then analyze the weighted graphs. In both cases, we put forward efficient algorithms to find the employer (employee) optimal balanced outcomes. Our algorithms are enabled by novel generalizations of the techniques proposed by Kleinberg and Tardos [6], which may be of independent interests.

Our work is also inspired by various previous work concerning stability and balancedness in different settings, such as [2, 4, 5, 7–9].

2 A MULTI-AGENT BARGAINING SOLUTION

In a cooperative game (N, v), define the minimum slack of set S over set T with respect to payoff vector γ to be $\sigma(S,T) = \min\{\gamma(U) - v(U) : U \subseteq N \setminus T, S \subseteq U\}$. Then we define that an outcome (B, γ) is balanced if and only if

- (B, γ) is in the core.
- $\forall S \in B$, there exists a non-trivial partition of S, $C = \{C_1, \ldots, C_k\}$ such that $\forall T \in C, \sigma(T, S \setminus T) = \min_{U \subsetneq S} \sigma(U, S \setminus U)$.

Intuitively, in a multi-agent coalition C, for any sub-coalition S, we assume every agent agrees that the cost of S deviating from C is $\sigma(S, C \setminus S)$. Then it can be modeled as a cooperative game on C, with coalition function $v'(S) = v_e - \sigma(S, C \setminus S)$ (for all $S \subsetneq C$). Here each coalition S is given an "incentive" value v_e if S breaks the current coalition C. Define v_e to be a *good incentive value* if the payoff of each agent remains unchanged regardless of whether they break the coalition C or not. Namely, the cooperative game has at least the following two core outcomes: $(\{C\}, \vec{0}), (B', \vec{0})$, where B' is a partition of C. Then if there exists a good incentive value, the coalition is balanced.

Then one can check the desiderata given in Section 1: It is defined on the core; If this definition is applied on social exchange networks, it is consistent with the original definition. Also, we will show this definition is computable in wage bargaining problem in the next section.

3 COMPUTE BALANCED OUTCOMES IN WAGE BARGAINING

An important and technical part of our contribution is to compute the balanced outcomes in wage bargaining problems. We first simplify the concept of balanced outcome in wage bargaining problem and introduce our algorithms which efficiently computes balanced outcomes in wage bargaining. Using organization similar to Kleinberg and Tardos' [6], we consider the easy case where the graph is unweighted and has a unique perfect matching first, which requires simpler notations and offers better insights. We then extend to the general case with weights. Compared to Kleinberg and Tardos' setting, our setting is more complicated. In particular, for each employer x, the employee y who brings smallest value to the employer need to be identified. To handle the difficulty, a kind of new structure called "loose chain" is invented by us to cooperate with the original structures: chains and free cycles (free cycles is not required in the unique matching case).

As a result, we prove the following theorem as our main result.

THEOREM 3.1. *There exists a balanced outcome for any wage bargaining problem. The set of balanced outcomes can also be efficiently generated. Moreover, the optimal outcomes for employers or employees can be computed in polynomial time.*

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