

The Rise and Fall of Complex Family Structures: Coalition Formation, Stability, and Power Struggle

Extended Abstract

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ABSTRACT

A complex family is a particular case of animal family structures that arises from cooperative breeding. This paper studies the problem of complex family formation from a game-theoretical perspective and proposes a characteristic function coalitional game. We investigate the stability of coalitions and provide theoretical bounds on the existence of complex families and the size of coalitions. Furthermore, we empirically examine the proposed framework and show that our results are consistent with the observed coalition formations, shedding light on the family compositions in the past.

KEYWORDS

Coalition formation; cooperative games; biologically-inspired applications

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1 INTRODUCTION

In wildlife populations, cooperative breeding represents an alternative to monogamous pairing and encompasses *complex families*, where offsprings receive additional care from group members other than their parents. A complex family, at its heart, is a heterogeneous multiagent system, consisting of a central pair of agents (breeders) and multiple secondary agents (helpers) that form a coalition. Contrary to non-cooperative games that require information about interactions between group members, a cooperative game theory posits a coalition as a basic unit and assigns to each coalition a payoff. Coalitional games provide a solid mathematical framework for analyzing the group formation strategies with respect to *all* agents, and have been extensively applied to various domains in artificial intelligence [10, 12].

In this paper, we develop a novel model of coalitional games for complex family formation. We identify the theoretical boundaries for the stability of the coalitions and provide empirical simulations. We base our analysis on the principles applied in previous models of family formation: the comparison of alternatives [5, 11, 17]. Moreover, coalitional game theory enables us to enhance this framework

by switching from a simple solitary choice to an interdependent set of choices. Our study is inspired by Arctic foxes from Mednyi Island. In contrast to mainland populations, this subspecies optionally form families with several non-breeding female helpers.

2 THE FAMILY FORMATION GAME

Let $P = \{m \cup N\}$ be a set of agents with m indicating an adult breeding male and $N = \{1, 2, 3, \dots, n\}$ denoting a set of adult females. Let $v : 2^P \rightarrow \mathbb{R}^{\geq 0}$ be a *characteristic function* for each coalition $C \subseteq P$. A *coalitional game* for family formation is then denoted by $G = (P, v)$ where $v(\emptyset) = 0$. A *family* is a coalition of agents that occupy a native home range, which can deliver a maximum payoff (capacity) of 1. Mature offspring of wild animals either disperse and try to form their own family (*migration*) or delay dispersal staying in the family (*philopatry*). Thus, each agent makes a choice: to stay with the family or to leave the home range.

We model the characteristic function as a function of the *number of females*, n ; *migration parameter*, h ; a *breeding pair utilization* of a home range capacity, a ; and a *diminishing factor* for the *marginal contribution* of each additional female to a family, $\gamma \geq 0$. The marginal contribution is limited by the maximum capacity of a home range; it is modeled as an exponentially decreasing function in line with other biological and financial processes [2, 3, 6, 13, 14]. Given a diminishing factor γ , $y(i) = e^{-\gamma(i-1)}$ is a function of *diminishing value* for i^{th} additional female.

We consider three types of coalitions: 1) a lone male: migrant; 2) a group of females: migrants; and 3) one male and at least 1 female: a family. The characteristic function of game G is written as:

$$v(C) = \begin{cases} h & \text{for } C = \{m\}, \\ h \cdot |C| & \text{for } \forall C \subseteq P : m \notin C \\ f(k) = a + (1-a) \cdot \frac{(\sum_{i=1}^k y(i)) - y(1)}{d - y(1)} & \text{for } \forall C \subseteq P : m \in C \end{cases} \quad (1)$$

where $a \in [0, 1]$, $h \in [0, 0.5]$, $k = |C| - 1$ is a number of females in C , and $d = \lim_{k \rightarrow +\infty} \sum_{i=1}^k y(i) = \frac{1}{1 - e^{-\gamma}}$ is the *normalization factor*.

Equation (1) satisfies the principles noted above: the maximum value of the family is $\lim_{k \rightarrow +\infty} f(k) = 1$; the value of a breeding pair is $f(1) = a$; and $\frac{f(k) - f(k-1)}{f(k-1) - f(k-2)} = e^{-\gamma}$, $\forall k > 2$, the relative contribution of a female compared to the preceding agents decreases exponentially. We also use the helper notations: $f(k) = 1 - (1-a) \cdot e^{-\gamma(k-1)}$ (since $\sum_{i=1}^k y(i)$ is a geometric series); $t = 1 - f(n) = (1-a) \cdot e^{-\gamma(n-1)}$: the *remaining capacity* of a home range.

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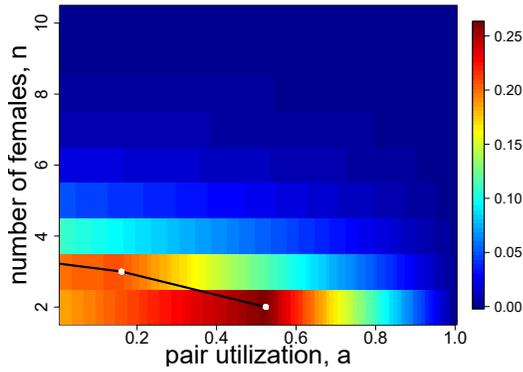


Figure 1: The space of the nonempty core given the parameters. Colors represent the value of \hat{h} – the maximum h value of nonempty core; the black line with white dots represent a^* – the value of a corresponding to $\max \hat{h}$ given n, γ .

3 STABILITY OF THE COMPLEX FAMILY

In coalitional games, we are interested in measuring whether agents will be willing to form a grand coalition (a coalition of a male and n females). If no sub-coalition has an incentive to break away from the grand coalition, then we say that the game has a *nonempty core* [7, 16].

THEOREM 3.1. *A family formation game with n, a, γ has a nonempty core only if $h \leq \hat{h}$, where \hat{h} :*

$$\hat{h} = \min \left(\frac{t}{d-1}, \frac{1-t}{n+1} \right) \quad (2)$$

Simply put, Theorem 3.1 sets the conditions for the parameters of the game that are necessary for the existence of the core.

We intentionally formulated the conditions of Theorem 3.1 in terms of the migration parameter, h . From the biological standpoint, parameter h defines the benefits of migration and represents the external environment to the family: availability of vacant home ranges nearby and migrant endangerment, which in turn depend on local population density. Local density is spatially and temporally variable, so does h – it is fixed for a given family for the given year but not for a population as a whole. In this context, \hat{h} (the maximum h corresponding to the nonempty core) implies the robustness of the grand coalition to the spatial variation and temporal changes. As \hat{h} rises, h can vary in a wider range without threats to the stability of the grand coalition: in other words, the ‘likelihood’ of complex family formation in a population increases.

LEMMA 3.2. *Given the number of females n and marginal contribution γ , there exists a threshold $a^*(n, \gamma)$ as follows:*

$$a^*(n, \gamma) = 1 - \frac{d-1}{e^{-\gamma(n-1)} \cdot (n+d)} \quad (3)$$

such that if $a > a^(n, \gamma)$ then $\hat{h} = \frac{t}{d-1}$, and if $a < a^*(n, \gamma)$ then $\hat{h} = \frac{1-t}{n+1}$.*

THEOREM 3.3. *Given n and γ , the highest likelihood of the complex family formation (maximum \hat{h}) is at $a = \max(a^*, 0)$.*

When a (the payoff to a coalition of a breeding pair) approaches 1, a home range can not afford more agents. If a reaches 0, then a breeding pair needs helpers to gain sufficient payoff (e.g. to protect cubs from intruders). As a increases, the contribution of the other females decreases, on the other hand the total value of the grand coalition rises. For $a < a^*$ the second (positive) effect outweighs the first (negative) and the likelihood of the complex family formation rises with a ; for $a > a^*$ this relationship is reversed.

4 EMPIRICAL RESULTS

We complement our analytical findings with empirical evaluation on a real-world case study: Arctic fox subspecies of Mednyi Island. We ran simulations to investigate the stability of complex families using the LP formulation of the core [16]. The parameter γ is set to 0.8 based on reports from Mednyi fox family structures [9]. Fig. 1 illustrates the space where the core is nonempty; the third dimension, color, represents \hat{h} – larger \hat{h} is interpreted as a higher likelihood of complex family formation of size n . When $a = a^*$, the likelihood is maximized for any given n, γ (see Theorem 3.3).

Fig. 1 clearly shows the optimum size for the stability of complex family, the number of females $n \in \{2, 3\}$ and the theoretical maximum size at $n \in \{5, 6\}$. For $a < 0.2$ the stability of families with $n = 2$ and 3 is nearly identical; hence both structures could coexist in a population as equals. Our analysis shows that when the pair utilization parameter a is in the range $0.5 < a < 0.7$, families with 2 females are two times more likely to appear than those with 3 females, and groups with more than 4 females are nearly absent. These findings interestingly justify the observed family structures of Mednyi Arctic fox subspecies [9]. In the mainland, which do not have complex families, a should be much higher ($a > 0.8$) corresponding to minor intruder pressure: the population density in the mainland is tens of times smaller than the Mednyi Island [9].

Lastly, we evaluated the Shapley value (for $\gamma = 0.8$ and $h = \hat{h}$) to measure the importance of agents in the family formation game. Intriguingly, a male agent’s power increases as n and a grow, and a female’s contribution can exceed that of the male’s when $n = 2$ and $n = 3$ for cases of $a < 0.50$ and $a < 0.25$, respectively.

5 CONCLUDING REMARKS

We provide a viable model for the analysis of coalitions in a variety of ecological multiagent systems. It enables researchers to study the model’s parameters in order to predict future ramifications of resource scarcity on animal behavior and estimate the sensitivity of populations to ecological changes. In particular, we can model the families of Mednyi foxes before a disease outbreak that wiped out 90% of the population in 1970–1980 [8]. The extra high population density before the outbreak implies low pair utilization ($a < 0.2$ as we believe), which corresponds to the family composition with the equal representation of groups with 2 and 3 females.

Lastly, our model may be adopted in other multiagent settings where team formation with agents of various types is key [1, 4]. For example, in most rescue operations, teams have a minimum threshold for their size and a set of skills with diminishing returns of additional members [15]. In the future, we plan to further generalize our model to other application domains and extend it to environments with uncertain parameters.

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