Robustness against Agent Failure in Hedonic Games

Extended Abstract

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ABSTRACT

In many real-world scenarios, *stability* is a key property in coalition formation to cope with uncertainty. In this paper, we propose a novel criterion that reshapes stability from robustness aspect. Specifically, we consider the problem of how stability can be maintained even after a small number of players leave the entire game, in the context of hedonic games. While one cannot guarantee the existence of robust outcomes with respect to most of the stability requirements, we identify several classes of friend-oriented and enemy-oriented games for which one can find a desired outcome efficiently. We also show that a symmetric additively hedonic game always admits an outcome that is individually stable and robust with respect to individual rationality.

KEYWORDS

Hedonic games; robustness; stability; agent failure

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1 INTRODUCTION

Hedonic coalition formation games (for short, hedonic games), introduced by Bogomolnaia and Jackson [6] and Banerjee et al. [5], provide an elegant framework to formulate coalition formation. In these games, each player has preferences over the coalitions to which she or he belongs, and desirable outcomes often correspond to *stable* partitions. The basic intuition behind stable partitioning is that group structures need to be robust under certain changes *within* the system; that is, outcomes must be immune to players' coalitional or individual deviations to other coalitions.

In many real-world scenarios, however, groups may encounter unexpected changes and challenges, imposed from the *outside* of the system. For instance, a certain country can go bankrupt and be enforced to leave a political alliance. In this respect, a group structure that satisfies a standard stability requirement can become immediately unstable due to unexpected circumstances. A case in point is a political coalition of three countries with one intermediate country connecting two other countries who are enemies to each other: if the intermediate player happens to disappear from the coalition, one cannot maintain the stability of the whole system.

In this paper, we propose a novel criterion that redefines stability from robustness aspect. We define an outcome to be *robust* with respect to a certain stability requirement α if removing any set of at most k players still preserves α. Besides the preceding example of a political alliance, there are several applications of hedonic games, such as project team formation [12], research team formation [1], and group activity selection [7], in which unexpected players' non-participation may severely affect stability of the system. To the best of our knowledge, however, no attempt has been ever made to connect two important considerations, robustness and stability. Our goal is to make the first step filling this gap.

A simple example of one player connecting two enemies shows impossibility in maintaining most of the stability properties. Given these negative results, we study a class of hedonic games that enable us to find robust outcomes efficiently. We focus on friend and enemy-oriented games, introduced by Dimitrov et al. [8], where players' preferences are succinctly encoded via the binary friendship relations, which we call *friendship graphs*. We show that for some topologies of the friendship-graph, we can efficiently decide whether the set of outcomes that are robust with respect to stability requirements is non-empty. We also consider the question of whether a minimum stability requirement, *individual rationality*, can be maintained while ensuring that an outcome of a game itself satisfies stronger stability properties. It turns out that when players have symmetric additively separable preferences, an individually stable partition which is robust with respect to individual rationality always exist.

Extended version An extended version is available on arXiv [10].

2 ROBUSTNESS IN HEDONIC GAMES

For a natural number $s \in \mathbb{N}$, we write $[s] = \{1, 2, \dots, s\}$. A hedonic game is defined as a pair $(N, (\succeq_i)_{i \in N})$ where $N = [n]$ is a finite set of *players* and each $\succeq_i$ is a preference over the subsets of N. For standard terminologies and notations of hedonic games, see [4]. We consider several established notions of stability, such as *core stability* (CR), *Nash stability* (NS), *individual stability* (IS), and

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		general graph	clique-tree	tree
Friend-oriented games	CR-robustness			poly time
	NS-robustness	poly time	poly time	poly time
	IS-robustness			poly time
	IS & IR-robustness	exists and polytime	exists and polytime	exists and polytime
Enemy-oriented games	CR-robustness			
	NS-robustness		poly time	poly time
	IS-robustness		poly time	poly time
	IS & IR-robustness	exists and polytime	exists and polytime	exists and polytime

Table 1: Overview of our complexity results. The top row corresponds to the restrictions on the friendship graphs. When no reference is given, the result follows from other results in the table. Problems corresponding to the blank cells remain open.

individual rationality (IR). Now we are in a position to make the definition of robustness more formal. For each $S \subseteq N$ and $i \in N$, we denote by $\succeq_i |_S$ the preference relation restricted to $\mathcal{N}_i \cap 2^S$.

Definition 2.1. Given a hedonic game $(N, (\succeq_i)_{i \in N})$, a natural number $k > 0$, and $\alpha \in$ {CR,NS,IS,IR}, a partition π of N is said to be α-robust under deletion of at most k players if π satisfies the property α, and for any $S \subseteq N$ with $|S| \leqslant k$, the partition $\pi_{-S} := \{ S' \setminus S \mid S' \in \pi \}$ still satisfies the property α in the subgame $(N \setminus S, (\succeq_i |_{N \setminus S})_{i \in N \setminus S})$. When k is clear from the context, we will simply call such partition α-robust.

By definition, if an outcome is α-robust under deletion of $k + 1$ players, then it is α-robust under deletion of any $\ell \leqslant k$ players. Fixing parameter k, the relations between the robustness concepts are the same as those among the corresponding stability concepts.

3 FRIEND-ORIENTED GAMES

We first focus on symmetric friend-oriented games [8], where there is an undirected graph describing players' friendship relations. Note that it is easy to construct stable outcomes in these games: dividing the players into connected components of the friendship graph results in a partition satisfying both core and Nash stability. If we additionally require robustness, however, no partition of a star of size three can preserve these stability properties. Given that we do not have an existence guarantee, a natural question is which structure of friendship graphs ensures existence and complexity guarantees of such outcomes. Specifically, we consider the case when the friendship graph is very well structured, i.e., *forest*, which often enables an efficient computation of desired outcomes (see e.g., [2, 9]). It turns out that in such settings, we can completely characterize the class of friendship graphs whose corresponding robust outcomes are non-empty: we show that when the friendship graph is a forest, there is a CR-robust/IS-robust partition if and only if the players can be partitioned into pairs of friends.

4 ENEMY-ORIENTED GAMES

We now move on to the case of enemy-oriented games [8], in which players have a strong aversion against enemies. As was necessary in friend-oriented games, a robust outcome of an enemy-oriented game must partition the players into completely balanced coalitions if the friendship graph is a forest. We consider a more general class of graph families with a tree-like structure, where the graph can be decomposed into the cliques forming a tree, known as a *clique-tree* [11]. For such topologies, one can decide the existence of an NS-robust/IS-robust outcome in polynomial time.

5 IR ROBUSTNESS

Our next question is the following: is it still possible to guarantee a minimum stability requirement, i.e., individual rationality, under deletion of players, while ensuring desirable property of the original partition? The answer is positive for individual stability when players have symmetric additively separable preferences. In these games, one can guarantee the existence of an individually stable partition that is IR-robust. In outline, the algorithm works as follows: starting with all-the singleton partition, we keep letting a player who has an IS-deviation deviate to his preferred coalition until there does not remain such a player. The algorithm correctly identifies an outcome that is individually stable and IR-robust due to the potential argument in [6] and the fact that each player only accepts a player for whom she has a non-negative utility.

6 DISCUSSION

There are several interesting questions for future work. Most obviously, while our main focus was on robustness against agents' non-participation, studying other types of robustness would be an important topic of research. For instance, one might want to consider sudden failure of agents' friendship relations, due to individual or political conflicts. There are also further classes subclasses of additively separable games that we have not considered in this paper, most notably fractional hedonic games [2, 3], which one can study from both existence and complexity aspects; in particular, it would be interesting to investigate whether a similar graph-theoretic characterization of friendship graphs that ensure the existence of stable outcomes can be obtained.

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