- $A^i_k = \{a^i_k, b^i_k, c^i_k\}$ for $i = 1, \ldots, 2^k$, and
- $A^i_j = \{a^i_j, b^i_j, c^i_j, \alpha^i_j, \beta^i_j, \gamma^i_j, \delta^i_j\}$ for $j = 0, \ldots, k-1, i = 1, \ldots, 2^j$.

Similar names of agents suggest that these agents are going to play the same role in the reduction. The preferences are designed in a way such that if there exists no 3-partition of R through sets in S, then there exists a unique best partition that assigns more than half of the agents a top-ranked coalition. Otherwise, there exists a partition that puts exactly all the other agents in one of their top coalitions. For the sets in the definition of the preferences, an arbitrary tie-breaking can be used to obtain strict preferences. We order the set R in an arbitrary but fixed way, say $R = \{r^1, \ldots, r^{|R|}\}$ and for a better understanding of the proof and the preferences, we label the agents $b^i_k = r^i$ for $i = 1, \ldots, |R|$. If we view the set of agents N as $k+1$ levels of agents, then the ground set R of the instance of X3C is identified with some specific agents in the top level k. Preferences of the agents are as follows.

- $\{a^i_k, b^i_k, c^i_k\} \succ_{a^i_k} \{a^i_k\}, i = 1, \ldots, 2^k$
- $\{a^i_j, \beta^i_j, \gamma^i_j\} \succ_{a^i_j} \{a^i_j, b^i_j, c^i_j\} \succ_{a^i_j} \{a^i_j\}, j = 0, \ldots, k-1, i = 1, \ldots, 2^j$
- $\{\{b^i_k, b^v_k, b^w_k\} : \{r^i, r^v, r^w\} \in S \text{ for some } 1 \leq v, w \leq |R|\} \succ_{b^i_k} \{a^i_k, b^i_k, c^i_k\} \succ_{b^i_k} \{b^i_k\}, i = 1, \ldots, |R|$
- $\{b^i_k\}, i = |R|+1, \ldots, 2^k$
- $\{b^i_j, c^{2i-1}_{j+1}, c^{2i}_{j+1}\} \succ_{b^i_j} \{a^i_j, b^i_j, c^i_j\} \succ_{b^i_j} \{b^i_j\}, j = 0, \ldots, k-1, i = 1, \ldots, 2^j$
- $\{a^i_j, b^i_j, c^i_j\} \succ_{c^i_j} \{c^i_j\}, j = 0, \ldots, k, i = 1, \ldots, 2^j$
- $\{\alpha^i_j, \beta^i_j\} \succ_{\alpha^i_j} \{\alpha^i_j\}, j = 0, \ldots, k-1, i = 1, \ldots, 2^j$
- $\{\beta^i_j, \gamma^i_j, a^i_j\} \succ_{\beta^i_j} \{\beta^i_j, \alpha^i_j\} \succ_{\beta^i_j} \{\beta^i_j\}, j = 0, \ldots, k-1, i = 1, \ldots, 2^j$
- $\{\gamma^i_j, \delta^i_j\} \succ_{\gamma^i_j} \{\gamma^i_j\}, j = 0, \ldots, k-1, i = 1, \ldots, 2^j$
- $\{\delta^i_j, \alpha^{2i-1}_{j+1}, \alpha^{2i}_{j+1}\} \succ_{\delta^i_j} \{\delta^i_j, \gamma^i_j\} \succ_{\delta^i_j} \{\delta^i_j\}, j = 0, \ldots, k-1, i = 1, \ldots, 2^j$

The structure of the flatmate game is illustrated in Figure 1 for the case $k = 3$. We will be particularly interested in coalitions of the types $\{a^i_j, b^i_j, c^i_j\}$, $\{\alpha^i_j, \beta^i_j\}$, and $\{\gamma^i_j, \delta^i_j\}$, which are indicated by undirected edges. These coalitions form the partition π^* of Lemma 4.10 that we need later to investigate for strong and mixed popularity in the respective reductions. The directed edges indicate that an agent at the tail of the arrow needs to form a coalition with the agent at the tip of the arrow in order to improve from her coalition of the above type. The set of agents consists of a binary tree of triangles depicted by the circular-shaped vertices. The important property of this tree is that whenever a coalition of the type $\{a^i_j, b^i_j, c^i_j\}$ gets dissolved, there can only be an improvement in popularity for the agents in A^i_j if they propagate changes in the partition upwards within this tree. This is achieved for agents b^i_j directly through the binary tree and for agents a^i_j with help of the auxiliary agents $\{\alpha^i_j, \beta^i_j, \gamma^i_j, \delta^i_j\}$ that are depicted as diamond-shaped vertices.

In the following lemma and theorem, we denote for any subset $M \subseteq N$ of agents and partitions π, π' of N, $\phi_M(\pi, \pi') = |N(\pi, \pi') \cap M| - |N(\pi', \pi) \cap M|$, that is, the popularity margin on a the subset M with respect to π and π'.

Lemma 4.10. *Let an instance (R, S) of X3C be given and define the corresponding flatmate game as above. Consider the partition $\pi^* = \{\{a^i_j, b^i_j, c^i_j\} : j = 0, \ldots, k, i = 1, \ldots, 2^j\} \cup \{\{\alpha^i_j, \beta^i_j\}, \{\gamma^i_j, \delta^i_j\} : j = 0, \ldots, k-1, i = 1, \ldots, 2^j\}$. Let $\pi \neq \pi^*$ be an arbitrary partition of agents distinct from π^*. Then $\phi(\pi^*, \pi) \geq 1$. In addition, if $c^1_0 \in N(\pi^*, \pi)$, then $\phi(\pi^*, \pi) \geq 3$ or $\{b^i_k : i = 1, \ldots, 2^k\} \subseteq N(\pi, \pi^*)$.*

Proof sketch. Let an instance (R, S) of X3C be given and define the corresponding flatmate game as above. Let π^* be defined as in the lemma and $\pi \neq \pi^*$ an other partition. We recursively define the following sets of agents: for $i = 1, \ldots, 2^k$, $T^i_k = A^i_k$ and for $j = k-1, \ldots, 0, i = 1, \ldots, 2^j$, $T^i_j = A^i_j \cup T^{2i-1}_{j+1} \cup T^{2i}_{j+1}$. The core of the proof is the following claim that can be proved by induction over $j = k, \ldots, 0$.

For every $i = 1, \ldots, 2^j$ holds: Assume there exists an agent $x \in T^i_j$ with $\pi(x) \neq \pi^*(x)$. Then $\phi_{T^i_j}(\pi^*, \pi) \geq 1$. If even $\pi(a^i_j) \neq \pi^*(a^i_j)$, then $\phi_{T^i_j}(\pi^*, \pi) \geq 3$ or $\{b^i_k : i = 1, \ldots, 2^k\} \cap T^i_j \subseteq N(\pi, \pi^*)$.

For the induction step, one essentially proves that changing the coalitions in A^i_j causes severe loss in popularity, unless we propagate changes to substructures via b^i_j or β^i_j. Clearly, the assertion of the lemma follows from the case $j = 0$. □

We are now ready to apply the lemma for the desired reductions.

Theorem 4.11. *Deciding whether there exists a strongly popular partition in flatmate games is coNP-hard, even if preferences are strict.*

Proof. The reduction is from X3C. Given an instance (R, S) of X3C, we define a hedonic game on agent set $N' = N \cup \{z\}$ where the agents N are as in the above construction with the identical preferences and $\{c^1_0, z\} \succ_z \{z\}$. Note that $|N'| = 3\sum_{j=0}^{k} 2^j + 4\sum_{j=0}^{k-1} 2^j + 1 = 10 \cdot 2^k - 6 = O(|R|)$ and the reduction is in polynomial time.

Consider the partition $\pi^* = \{\{a^i_j, b^i_j, c^i_j\} : j = 0, \ldots, k, i = 1, \ldots, 2^j\} \cup \{\{\alpha^i_j, \beta^i_j\}, \{\gamma^i_j, \delta^i_j\} : j = 0, \ldots, k-1, i = 1, \ldots, 2^j\} \cup \{\{z\}\}$. It follows directly from Lemma 4.10 that π^* is popular and hence there exists a strongly popular partition if and only if π^* is strongly popular. We will prove that this is the case if and only if the instance of X3C is a 'no'-instance.

Assume that there exists no 3-partition of R through sets in S and let an arbitrary partition $\pi \neq \pi^*$ be given. Then there exists an agent $x \in N$ with $\pi(x) \neq \pi^*(x)$ and it follows from Lemma 4.10 that $\phi(\pi^*, \pi) \geq \phi_N(\pi^*, \pi) - 1 \geq 3 - 1 = 2$. Hence, π^* is strongly popular.

Conversely, assume that there exists a 3-partition $S' \subseteq S$ of R. Define $\pi = \{\{b^v_k, b^w_k, b^x_k\} : \{v, w, x\} \in S'\} \cup \{\{b^i_k\} : i = |R|+1 \ldots, 2^k\} \cup \{\{\delta^i_{k-1}, a^{2i-1}_k, a^{2i}_k\} : i = 1, \ldots, 2^{k-1}\} \cup \{\{b^i_j, c^{2i-1}_{j+1}, c^{2i}_{j+1}\}, \{\delta^i_j, \alpha^{2i-1}_{j+1}, \alpha^{2i}_{j+1}\}, \{a^i_j, \beta^i_j, \gamma^i_j\} : j = 1, \ldots, k-1, i = 1, \ldots, 2^j\} \cup \{\{\alpha^1_0\}, \{z, c^1_0\}\}$. It is easily checked that $\phi(\pi, \pi^*) = 0$.

Indeed, $N(\pi, \pi^*) = \{b^i_k : i = 1, \ldots, 2^k\} \cup \{\beta^i_j, \delta^i_j, a^i_j : j = 0, \ldots, k-1, i = 1, \ldots, 2^j\} \cup \{z\}$. Therefore, $|N(\pi, \pi^*)| = 2^k + 4\sum_{j=1}^{k-1} 2^j + 1 = 5 \cdot 2^k - 3 = \frac{1}{2}|N'|$. Hence, $\phi(\pi, \pi^*) \geq 0$ and equality follows from popularity of π^*. Therefore, there exists no strongly popular partition. □

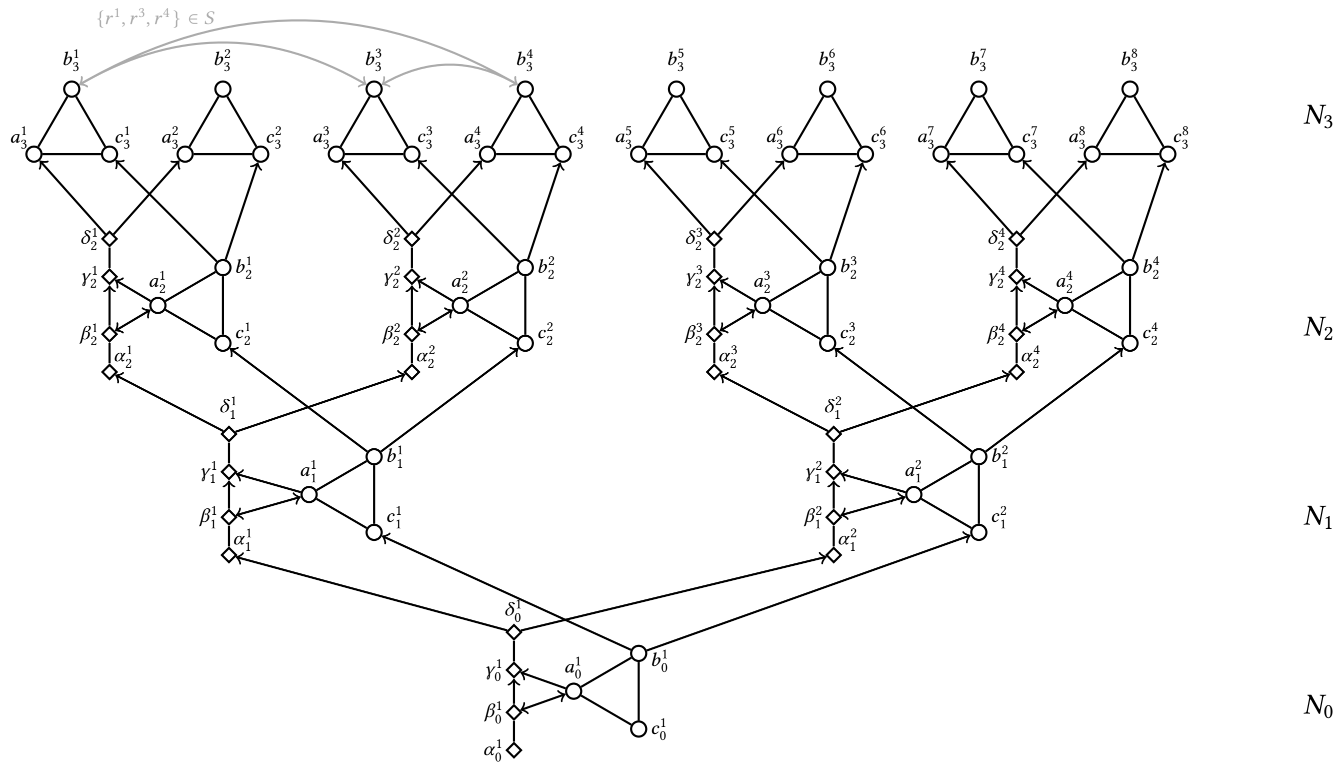

Figure 1: Schematic of the reduction for flatmate games with strict preferences. There is an edge between two agents if they are in the coalition π^* defined in Lemma 4.10. Directed edges indicate improvements from π^*. The gray edges suggest a 3-elementary set in S.

A similar reduction as in Theorem 4.11 works also for mixed popularity. However, we need two auxiliary agents to control the switch between a strongly popular and non-popular partition.

Theorem 4.12. *Computing a mixed popular partition in flatmate games is NP-hard, even if preferences are strict.*

To conclude the section, we deal with the problem of verifying whether a given partition is popular or strongly popular. Hardness of verifying popular partitions in flatmate games is shown by a complicated reduction from E3C. We have gadgets for elements in S and control the switch between 'yes' and 'no' instances by means of a binary tree. For a simpler, yet weaker hardness result, this tree could be contracted into a single vertex, but only at the expense of having to allow for coalitions of more than three agents.

Theorem 4.13. *Verifying whether a given partition in a flatmate game with strict preferences is popular is coNP-complete.*

For strong popularity, we obtain the same result.

Theorem 4.14. *Verifying whether a given partition in a flatmate game is strongly popular is coNP-complete, even if preferences are strict.*

Proof. In the proof of Theorem 4.11, the partition π^* is strongly popular if, and only if, (R, S) is a 'no'-instance of X3C. □

4.2.3 ***Globally Ranked Preferences.*** A natural question that arises after hardness results have been established is whether there are meaningful preference restrictions under which these results do not hold. In many cases, hardness breaks down when assuming that preference profiles adhere to certain structural restrictions. One such preference restriction that has been considered in the domain of coalition formation is that there exists one common global ranking $\succsim$ of all coalitions in $2^N \setminus \{\emptyset\}$ and each individual preference relation $\succsim_i$ is the restriction of $\succsim$ to $\mathcal{N}_i$. It is known that under these *globally ranked preferences*, every roommate game admits a stable matching, which can furthermore be efficiently computed [1]. Since every stable matching also happens to be popular (see Section 2), this implies that computing popular matchings in roommates games, which was recently shown to be NP-hard [19, 22, 26], becomes tractable under globally ranked preferences.

By contrast, all hardness results shown in Section 4.2.2 hold even when preferences are globally ranked. This confirms the robustness of these results and underlines the crucial difference between settings with coalitions of size 2 and coalitions of size 3.

4.3 Cardinal Hedonic Games

Important subclasses of hedonic games that admit succinct representations are based on cardinal utility functions. For one, there are *additively separable hedonic games* [10], where the utility that an

	weak preferences				*strict preferences*			
	PO	mPop	sPop	Pop	PO	mPop	sPop	Pop
IRLC	↑	↑	↑	↑	in P	↑	↑	↑
Flatmates	NP-h.[a]	NP-h. (Th. 4.9)	NP-h. (Th. 4.11)	↑	↓	NP-h. (Th. 4.12)	NP-h. (Th. 4.11)	↑
Roommates	in P[b]	in P (Th. 4.5)	in P (Cor. 4.8)	↑	↓	in P (Th. 4.5)	in P[d]	NP-h.[g]
Marriage	↓	↓	↓	NP-h.[e]	↓	↓	↓	in P[f]
Housing	↓	↓	↓	in P[c]	in P	in P[h]	in P	in P[c]

Table 1: Complexity of finding partitions in ordinal hedonic games. New results are highlighted in gray and implications are marked by gray arrows. NP-hardness of computing a popular or strongly popular partition always follows by a Turing reduction from the existence problem. Whenever computing a mixed popular partition is NP-hard, then verifying a deterministic partition is coNP-complete.

[a]: Aziz et al. [4, Th. 5], [b]: Aziz et al. [4, Th. 7], [c]: Abraham et al. [2, Th. 3.9], [d]: Biró et al. [9, Th. 6], [e]: Biró et al. [9, Th. 11], Cseh et al. [18, Th. 2], [f]: Gärdenfors [24, Th. 3], [g]: Gupta et al. [26, Th. 1.1], Faenza et al. [22, Th. 4.6], Cseh and Kavitha [19, Th. 2], [h]: Kavitha et al. [31, Th. 2]

agent associates with a coalition is the sum of utilities he ascribes to each member of the coalition. On the other hand, there are *fractional hedonic games* [3], where the sum of utilities is divided by the number of agents contained in the coalition.

In the following, let $v_i(j)$ denote the utility that agent i associates with agent j. A hedonic game $(N, \succsim)$ is an *additively separable hedonic game* (ASHG) if there is $(v_i(j))_{i,j \in N}$ that for every agent i, the preferences $\succsim_i$ are induced by the cardinal utilities given by $v(S) = \sum_{j \in S} v_i(j)$, for $S \subseteq N$. The hedonic game $(N, \succsim)$ is a *fractional hedonic game* (FHG) if there exists $(v_i(j))_{i,j \in N}$ such that for every agent i, the preferences $\succsim_i$ are induced by the cardinal utilities given by $v(S) = (\sum_{j \in S} v_i(j))/|S|$, for $S \subseteq N$. We focus on *symmetric* ASHGs and FHGs, i.e., games for which $v_i(j) = v_j(i)$ for all $i, j \in N$.

All hardness results in this section are obtained by rather involved reductions from E3C.

Theorem 4.15. *Checking whether there exists a popular partition in a symmetric ASHG is NP-hard.*

The verification problem for ASHGs turns out to be coNP-complete. The proof of Theorem 4.16 is simpler and holds for a more restricted class of games than the proof by Aziz et al. [5].

Theorem 4.16. *Checking whether a given partition in a symmetric ASHG is popular is coNP-complete.*

The reductions for mixed and strong popularity on ASHGs rely on a similar idea as for flatmate games with strict preferences. We find a graph that satisfies similar properties as the flatmate game considered in Lemma 4.10. This graph is used for the next four results.

Theorem 4.17. *Checking whether there exists a strongly popular partition in a symmetric ASHG is coNP-hard.*

Theorem 4.18. *Verifying whether a given partition in a symmetric ASHG is strongly popular is coNP-complete.*

Theorem 4.19. *Computing a mixed popular partition in a symmetric ASHG is NP-hard.*

We even obtain coNP-hardness of the existence of popular partitions.

Theorem 4.20. *Checking whether there exists a popular partition in a symmetric ASHG is coNP-hard.*

Theorem 4.21. *Checking whether there exists a popular partition in a symmetric FHG is NP-hard, even if all weights are non-negative.*

The hardness proof for the verification problem for FHGs is a more involved version of the proof for ASHGs.

Theorem 4.22. *Checking whether a given partition in a symmetric FHG is popular is coNP-complete, even if all weights are non-negative and the underlying graph is bipartite.*

The graphs used in the proof of Theorem 4.22 have girth 6. This is in contrast to the polynomial-time algorithm by Aziz et al. [3] for computing the core on FHGs with girth at least 5.

5 CONCLUSION

We have investigated the computational complexity of finding and recognizing popular, strongly popular, and mixed popular partitions in various types of ordinal hedonic games (see Table 1) and cardinal hedonic games. Two important factors that govern the complexity of computing these partitions in ordinal hedonic games are whether preferences may contain ties and whether coalitions of size 3 are allowed. When preferences are weak, computing mixed popular and strongly popular partitions is only difficult for representations for which we cannot even compute Pareto optimal partitions efficiently. For strict preferences, however, Pareto optimal partitions can be found efficiently while computing mixed popular and strongly popular partitions remains intractable, even when preferences are globally ranked. Our positive results are obtained via a single linear programming approach that unifies a number of existing results and exploits the relationships between the different types of popularity. Finally, we complete the picture by providing a variety of results showing the intractability of popular, strongly popular, and mixed popular partitions in ASHGs and FHGs.

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REFERENCES

[1] D. J. Abraham, A. Leravi, D. F. Manlove, and G. O'Malley. 2007. The stable roommates problem with globally-ranked pairs. In *Proceedings of the 3rd International Workshop on Internet and Network Economics (WINE) (Lecture Notes in Computer Science (LNCS))*, Vol. 4858. Springer-Verlag, 431–444.

[2] D. K. Abraham, R. W. Irving, T. Kavitha, and K. Mehlhorn. 2007. Popular matchings. *SIAM J. Comput.* 37, 4 (2007), 1030–1034.

[3] H. Aziz, F. Brandl, F. Brandt, P. Harrenstein, M. Olsen, and D. Peters. 2019. Fractional Hedonic Games. *ACM Transactions on Economics and Computation* 7, 2 (2019).

[4] H. Aziz, F. Brandt, and P. Harrenstein. 2013. Pareto Optimality in Coalition Formation. *Games and Economic Behavior* 82 (2013), 562–581.

[5] H. Aziz, F. Brandt, and H. G. Seedig. 2013. Computing Desirable Partitions in Additively Separable Hedonic Games. *Artificial Intelligence* 195 (2013), 316–334.

[6] H. Aziz, F. Brandt, and P. Stursberg. 2013. On Popular Random Assignments. In *Proceedings of the 6th International Symposium on Algorithmic Game Theory (SAGT) (Lecture Notes in Computer Science (LNCS))*, Vol. 8146. Springer-Verlag, 183–194.

[7] H. Aziz and R. Savani. 2016. Hedonic Games. In *Handbook of Computational Social Choice*, F. Brandt, V. Conitzer, U. Endriss, J. Lang, and A. D. Procaccia (Eds.). Cambridge University Press, Chapter 15.

[8] C. Ballester. 2004. NP-completeness in hedonic games. *Games and Economic Behavior* 49, 1 (2004), 1–30.

[9] P. Biró, R. W. Irving, and D. F. Manlove. 2010. Popular Matchings in the Marriage and Roommates Problems. In *Proceedings of the 7th Italian Conference on Algorithms and Complexity (CIAC)*. 97–108.

[10] A. Bogomolnaia and M. O. Jackson. 2002. The Stability of Hedonic Coalition Structures. *Games and Economic Behavior* 38, 2 (2002), 201–230.

[11] F. Brandl and F. Brandt. 2019. Arrovian Aggregation of Convex Preferences. *Econometrica* (2019). Forthcoming.

[12] F. Brandl, F. Brandt, and J. Hofbauer. 2017. Random Assignment with Optional Participation. In *Proceedings of the 16th International Conference on Autonomous Agents and Multiagent Systems (AAMAS)*. IFAAMAS, 326–334.

[13] F. Brandl, F. Brandt, and H. G. Seedig. 2016. Consistent Probabilistic Social Choice. *Econometrica* 84, 5 (2016), 1839–1880.

[14] F. Brandl, F. Brandt, and C. Stricker. 2018. An Analytical and Experimental Comparison of Maximal Lottery Schemes. In *Proceedings of the 27th International Joint Conference on Artificial Intelligence (IJCAI)*. IJCAI, 114–120.

[15] F. Brandl and T. Kavitha. 2018. Two Problems in Max-Size Popular Matchings. *Algorithmica* 81, 7 (2018), 2738–2764.

[16] F. Brandt, J. Hofbauer, and M. Suderland. 2017. Majority Graphs of Assignment Problems and Properties of Popular Random Assignments. In *Proceedings of the 16th International Conference on Autonomous Agents and Multiagent Systems (AAMAS)*. IFAAMAS, 335–343.

[17] Á. Cseh. 2017. Popular Matchings. In *Trends in Computational Social Choice*, U. Endriss (Ed.). AI Access, Chapter 6.

[18] Á. Cseh, C.-C. Huang, and T. Kavitha. 2015. Popular matchings with two-sided preferences and one-sided ties. In *Proceedings of the 42nd International Colloquium on Automata, Languages, and Programming (ICALP) (Lecture Notes in Computer Science (LNCS))*, Vol. 9134. 367–379.

[19] Á. Cseh and T. Kavitha. 2018. Popular Matchings in Complete Graphs. In *Proceedings of the 37th IARCS Annual Conference on Foundations of Software Technology and Theoretical Computer Science (FSTTCS)*.

[20] J. H. Drèze and J. Greenberg. 1980. Hedonic Coalitions: Optimality and Stability. *Econometrica* 48, 4 (1980), 987–1003.

[21] J. Edmonds. 1965. Maximum Matching and a Polyhedron with 0,1-vertices. *Journal of Research of the National Bureau of Standards B* 69 (1965), 125–130.

[22] Y. Faenza, T. Kavitha, V. Power, and X. Zhang. 2019. Popular Matchings and Limits to Tractability. In *Proceedings of the 30th Annual ACM-SIAM Symposium on Discrete Algorithms (SODA)*. 2790–2809.

[23] P. C. Fishburn. 1984. Probabilistic Social Choice Based on Simple Voting Comparisons. *Review of Economic Studies* 51, 4 (1984), 683–692.

[24] P. Gärdenfors. 1975. Match Making: Assignments based on bilateral preferences. *Behavioral Science* 20, 3 (1975), 166–173.

[25] M. Grötschel, L. Lovász, and A. Schrijver. 1981. The Ellipsoid Method and its Consequences in Combinatorial Optimization. *Combinatorica* 1 (1981), 169–197.

[26] S. Gupta, P. Misra, S. Saurabh, and M. Zehavi. 2019. Popular Matching In Roommates Setting is NP-hard. In *Proceedings of the 30th Annual ACM-SIAM Symposium on Discrete Algorithms (SODA)*. 2810–2822.

[27] C.-C. Huang and T. Kavitha. 2011. Popular Matchings in the Stable Marriage Problem. In *Proceedings of the 38th International Colloquium on Automata, Languages, and Programming (ICALP) (Lecture Notes in Computer Science (LNCS))*, Vol. 6755. 666–677.

[28] C.-C. Huang and T. Kavitha. 2017. Popularity, Mixed Matchings, and Self-Duality. In *Proceedings of the 28th Annual ACM-SIAM Symposium on Discrete Algorithms (SODA)*. 2294–2310.

[29] R. M. Karp. 1972. Reducibility among Combinatorial Problems. In *Complexity of Computer Computations*, R. E. Miller and J. W. Thatcher (Eds.). Plenum Press, 85–103.

[30] T. Kavitha. 2014. A Size-Popularity Tradeoff in the Stable Marriage Problem. *SIAM J. Comput.* 43, 1 (2014), 52–71.

[31] T. Kavitha, J. Mestre, and M. Nasre. 2011. Popular mixed matchings. *Theoretical Computer Science* 412, 24 (2011), 2679–2690.

[32] T. Kavitha and M. Nasre. 2009. Optimal popular matchings. *Discrete Applied Mathematics* 157 (2009), 3181–3186.

[33] L. Khachiyan. 1979. A Polynomial Algorithm in Linear Programming. *Soviet Mathematics Doklady* 20 (1979), 191–194.

[34] T. Király and Z. Mészáros-Karkus. 2017. Finding strongly popular b-matchings in bipartite graphs. *Electronic Notes in Discrete Mathematics* 61 (2017), 735–741.

[35] M. Mahdian. 2006. Random popular matchings. In *Proceedings of the 7th ACM Conference on Electronic Commerce (ACM-EC)*. 238–242.

[36] D. F. Manlove. 2013. *Algorithmics of Matching Under Preferences*. World Scientific Publishing Company.

[37] R. M. McCutchen. 2008. The Least-Unpopularity-Factor and Least-Unpopularity-Margin Criteria for Matching Problems with One-Sided Preferences. In *Proceedings of the 8th Latin American Conference on Theoretical Informatics (LATIN) (Lecture Notes in Computer Science (LNCS))*, Vol. 4957. 593–604.

[38] J. von Neumann. 1928. Zur Theorie der Gesellschaftspiele. *Math. Ann.* 100, 1 (1928), 295–320.

[39] J. von Neumann and O. Morgenstern. 1947. *Theory of Games and Economic Behavior* (2nd ed.). Princeton University Press.