

where $e \in \langle k, \dots, f \rangle$. Recall that d^B prioritises shorter length delegation chains (see definition 8). We show that voter k has a shorter delegation chain available that does not include j , i.e. there exists a C'_k such that $|C'_k| < |C_k|$ and $j \notin C'_k$. Let $C'_k = \langle k, \dots, e, g, \dots, i \rangle$. According to d^B , the delegation chain used to assign j 's guru, $\langle j, \dots, e, g, \dots, i \rangle$, is shorter or equal in length to any other alternative, thus $|\langle j, \dots, e, g, \dots, i \rangle| \leq |\langle j, \dots, l \rangle|$. Observe that

$$\begin{aligned} |\langle g, \dots, i \rangle| &< |\langle j, \dots, e, g, \dots, i \rangle| \leq |\langle j, \dots, l \rangle| \Rightarrow \\ |\langle k, \dots, e \rangle| + |\langle g, \dots, i \rangle| &< |\langle k, \dots, e \rangle| + |\langle j, \dots, l \rangle|. \end{aligned}$$

Since $e \in \langle k, \dots, f \rangle$, we can rewrite the previous as

$$|\langle k, \dots, e \rangle| + |\langle g, \dots, i \rangle| < |\langle k, \dots, f \rangle| + |\langle j, \dots, l \rangle|.$$

Therefore, rule d^B should use C'_k to assign k 's guru. However, since $j \notin C'_k$, the assumption is contradicted.

The contradictions of both (a) and (b) prove this lemma. \square

THEOREM 5.7. *Given the majority rule $f \in \bar{F}$, guru participation is guaranteed to hold when using the breadth-first delegation rule d^B .*

PROOF. By Observation 2, given voters j and k in the delegating electorate, if a voter k does not delegate through j , then k 's assigned guru (if any) is the same even if j abstained. By Lemma 5.6, if a voter k delegates through j , then the guru of k is the same as the guru of j . Combining the above cases, we show that (regardless of k delegating through j or not), whenever a voter j joins the delegating electorate and is assigned to a guru i , then i is the only casting voter who increases the number of times she becomes a guru. Since also $f \in \bar{F}$, then Observation 1 holds, meaning that the breadth-first delegation rule d^B is guaranteed to satisfy guru participation. \square

6 CONCLUSION AND FUTURE WORK

In this paper, we discuss the depth-first and the breadth-first delegation rule proving that only the latter has the desirable property that every guru weakly prefers receiving delegated voting rights under the majority rule with two alternatives. The immediate future questions that arise are to investigate what holds for more than two alternatives in this model or for other voting rules that satisfy cast participation such as approval voting (Felsenthal [2010]). However, there could be delegation rules that satisfy other interesting properties which improve the concept of liquid democracy. Towards this path, we note that one of the main issues that current liquid democracy implementations suffer from, is that large parts of an electorate might end up being represented by only a small subset of gurus (Kling et al. [2015]). Since the breadth-first delegation rule

favours keeping delegated voting rights close to their origin, could this issue be resolved by using this rule? Other interesting future directions are investigating guru participation with voting rules that do not satisfy cast participation, relaxing the assumption of strict personal rankings over voters, and analysing other types of participation.

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