optimum of the following set: $\left\{\frac{-x_{s,p}}{t_{s,p}}\right\}_{s\in S, p\in P_s}$. The fairest feasible funding allocation can be achieved via Algorithm 2, which runs a series of linear programs.

Theorem 6.1. *Given a feasible matching, the fairest funding allocation can be computed in time* $O(|S|^2|P|^2)LP(O(|S|\cdot|P|))$, *where LP refers to the running time of the linear programming algorithm used.*

Proof. The algorithm solves a series of linear programs with $O(|S|\cdot|P|)$ constraints. The while loop iterates at most $|T| \leq |S||P|$ times, as the algorithm adds at least one element to the set T_{tight} in every iteration. The for loop also iterates at most $|T|$ times. Thus, the overall complexity is $O(|S|^2|P|^2)LP(O(|S|\cdot|P|))$, where LP refers to the running time of the linear programming algorithm used. Since linear programs can be solved in polynomial time, the fairest funding allocation can also be computed in polynomial time. □

7 CONCLUSION

We presented a novel matching model that captures many real-world scenarios. For the model, we presented a compelling solution that is polynomial-time and satisfies stability and fairness properties. Several directions and problems arise as a result of our study. Our approach to finding a fair budget allocation was to first compute a weakly stable matching and then find the fairest possible budget allocation. It will be interesting to explore a fair outcome that is fairest in some global sense across all weakly stable matchings. We showed that the algorithm we consider is not strategyproof for applicants. It is open whether there exists an algorithm that is strategyproof and satisfies weak stability. The problem has been open even for the abstract setting of Kamada and Kojima [15].

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