value [22], for every agent $j \in \mathcal{S}_i$ its marginal contribution to a coalition $\mathcal{S} \subseteq \mathcal{N} - \{i\}$ is given as follows:

$$u(\mathcal{S} \cup \{j\}) - u(\mathcal{S}) = c_i^2 - [c_i - (|S|+1)]^2 - [c_i^2 - (c_i - |S|)^2] =$$
$$= (c_i-|S|)^2-(c_i-|S|-1)^2 = (c_i-|S|+c_i-|S|-1)(c_i-|S|-c_i+|S|+1) =$$
$$= [2c_i - 2|S| - 1]$$

Given a set of size $|\mathcal{S}_i| - 1$, the number of its subsets of size ℓ is $\binom{|\mathcal{S}_i|-1}{\ell}$. Thus, for each $\mathcal{S} \subseteq \mathcal{S}_i - \{j\}$ such that $|S| = \ell$, we infer:

$$w_{\mathcal{S}} \cdot \binom{|\mathcal{S}_i|-1}{\ell} = \frac{\ell!(|\mathcal{S}_i| - \ell - 1)!}{|\mathcal{S}_i|!}\binom{|\mathcal{S}_i|-1}{\ell} =$$
$$= \frac{\ell!(|\mathcal{S}_i| - \ell - 1)!}{|\mathcal{S}_i|!} \cdot \frac{(|\mathcal{S}_i|-1)!}{\ell!(|\mathcal{S}_i| - \ell - 1)!} = \frac{1}{|\mathcal{S}_i|}$$

Accordingly, we have that:

$$\phi_j(\mathcal{S}_i, u) = \sum_{\mathcal{S}\subseteq\mathcal{S}_i-\{j\}} w_{\mathcal{S}}[u(\mathcal{S}\cup\{j\})-u(\mathcal{S})] = \frac{1}{|\mathcal{S}_i|}\sum_{\ell=0}^{|\mathcal{S}_i|-1}[2c_i-2\ell-1] =$$
$$= \frac{|\mathcal{S}_i|}{|\mathcal{S}_i|}2c_i - \frac{2}{|\mathcal{S}_i|}\sum_{\ell=0}^{|\mathcal{S}_i|-1}\ell - \frac{|\mathcal{S}_i|}{|\mathcal{S}_i|} = 2C_i - \frac{2}{|\mathcal{S}_i|}\cdot\frac{|\mathcal{S}_i|}{2}[0+|\mathcal{S}_i|-1]-1 =$$
$$= 2c_i - |\mathcal{S}_i| + 1 - 1 = \boxed{2c_i - |\mathcal{S}_i|}$$

□

The following corollary gives us an expression for the relative efficiency of each restricted cooperative game's grand coalition.

Corollary 4.3. *Let $(\mathcal{N} = A^F \cup A^D, v, \mathcal{CS})$ be a consensus-prevention game with a static coalition structure $\mathcal{CS} = \{\mathcal{S}_1, \ldots, \mathcal{S}_\xi\}$. Let $(\mathcal{S}_i, v|_{\mathcal{S}_i})$ be a restricted cooperative game, for some $\mathcal{S}_i \in \mathcal{CS}$. The relative efficiency of this game's grand coalition is:*

$$v|_{\mathcal{S}_i}(\mathcal{S}_i) = 2c_i|\mathcal{S}_i| - |\mathcal{S}_i|^2$$

Proof. For each coalition $\mathcal{S}_i \in \mathcal{CS}$:

$$v|_{\mathcal{S}_i}(\mathcal{S}_i) = \sum_{j\in\mathcal{S}_i}\phi_j(\mathcal{S}_i, v|_{\mathcal{S}_i}) = \sum_{j\in\mathcal{S}_i}[2c_i-|\mathcal{S}_i|] = \boxed{2c_i(\xi)|\mathcal{S}_i| - |\mathcal{S}_i|^2}$$

□

Given that the agents are aiming at maximizing the disagreement measure, the following theorem shows that *maximizing the disagreement measure is equivalent to maximizing the coalition structure's value.*

Theorem 4.4. *Let $(\mathcal{N} = A^F \cup A^D, v, \mathcal{CS})$ be a consensus-prevention game with a static coalition structure $\mathcal{CS} = \{\mathcal{S}_1, \ldots, \mathcal{S}_\xi\}$. Then, the disagreement measure reaches its maximum **if and only if** the value of the coalition structure reaches its maximum, i.e., it equals to:*

$$v(\mathcal{CS}) = \sum_{i=1}^{\xi} c_i^2 = \xi r^2 + 2qr + q$$

Proof. $\Leftarrow$ The value of the coalition structure is given by the following expression: $v(\mathcal{CS}) = \sum_{\mathcal{S}_i\in\mathcal{CS}} v(\mathcal{S}_i)$. Thus, the maximizing the coalition structure's value is equivalent to the maximizing the payoff of each coalition $\mathcal{S}_i \in \mathcal{CS}$. The first and second derivatives (respectively) of the characteristic function given in Definition 4.1 with respect to the size of the a coalition $\mathcal{S} \subseteq \mathcal{S}_i$ are as follows: $v'(\mathcal{S}) = 2(c_i - |\mathcal{S}|), v''(\mathcal{S}) = -2$. The characteristic function reaches an extremum value when its first derivative equals to zero: $2(c_i - |\mathcal{S}|) = 0 \Rightarrow |\mathcal{S}| = c_i$. Since $v''(\mathcal{S}) = -2 < 0$, for each coalition $\mathcal{S}_i \in \mathcal{CS}$ such that $|\mathcal{S}_i| = c_i$, the coalition receives its maximum payoff, which equals to $v(\mathcal{S}_i) = c_i^2$. As in Subsection 3.2, this also guarantees the maximization of the disagreement measure.

$\Rightarrow$ Regarding Equation 2, eventually $|\mathcal{S}_i| = c_i$ for every $1 \le i \le \xi$. Thus, for each agent $j \in \mathcal{S}_i$ the Shapley value $\phi(\mathcal{S}_i, v|_{\mathcal{S}_i})$ assigns the payoff $\phi_j(\mathcal{S}_i, v|_{\mathcal{S}_i}) = c_i$. Hence, the relative efficiency of each restricted cooperative game's grand coalition becomes $v|_{\mathcal{S}_i}(\mathcal{S}_i) = v(\mathcal{S}_i) = c_i^2$. In particular, the coalition structure's value reaches the following: $v(\mathcal{CS}) = \sum_{\mathcal{S}_i\in\mathcal{CS}} v(\mathcal{S}_i) = \sum_{i=1}^{\xi} c_i^2$. Indeed, considering the proof of the previous direction, it really is the coalition structure's maximum value. Furthermore, as mentioned in Subsection 3.2, we have that:

$$v(\mathcal{CS}) = \sum_{i=1}^{\xi} c_i^2 = \sum_{i=1}^{\xi-q} c_i^2 + \sum_{i=\xi-q+1}^{\xi} c_i^2 = \sum_{i=1}^{\xi-q} r^2 + \sum_{i=\xi-q+1}^{\xi}(r+1)^2 =$$
$$= (\xi-q)r^2 + q(r+1)^2 = (\xi-q)r^2 + qr^2 + 2qr + q = \boxed{\xi r^2 + 2qr + q}$$

□

Consequently from Theorem 4.4, we infer that **maximizing the disagreement measure is an NP-hard process**.

5 Consensus-Prevention - Problem Definition

The ***consensus-prevention problem*** is formally defined as follows:

Definition 5.1. (*Consensus-Prevention Problem*) Given a group of flocking agents $A^F := \{a_0, ..., a_{k-1}\}$, a group of diverting agents $A^D := \{a_k, ..., a_{n-1}\}$, and a set of desired orientations after convergence - $\Theta := \{\alpha_1, ..., \alpha_\xi\}$. We are aiming to find a partition $P = (V_1, \ldots, V_\xi)$ of the flocking neighbors graph at time step 0, $\mathcal{G}(0) = (\mathcal{V}, \mathcal{E}(0))$, and initial locations for the diverting agents, that will guarantee that flocking agents associated with the cluster V_i will converge to the orientation α_i (if possible), while maximizing the disagreement measure.

In general, the consensus-prevention problem requires edge operations (both additions and deletions) of the flocking neighbors graph, thus making it a variant of the **CLUSTER-EDITING** problem, which is known to be **NP-complete** [21]. Therefore, we restrict the analysis to a case in which the problem is solvable in **polynomial** time, and a partition can be readily computed, with less computational expense. Henceforth, we examine the case in which the number of connected components in the flocking neighbors graph at time step 0 is at least the number of desired orientations after convergence ($\eta \ge \xi$). First, we deal with determining the initial placements of the diverting agents in a flock that guarantees consensus-prevention. When the number of desired orientations is fixed, we prove that this can be done in **polynomial** time (Section 6.1). When also considering the *maximization* of the disagreement measure, we need an additional and quite restrictive property to be satisfied for the problem to be solvable in polynomial time: The partition is obtained by only aggregating connected components in the flocking neighbors graph at time step 0.

5.1 Diverting Agents' Initial Placements - Problem Definition

We first define the following two problems:

Definition 5.2. **DIP** (**D**iverting agents **I**nitial **P**lacements) - The input is as follows: (1) The initial placements of the k flocking agents; (2) The desired orientations after convergence: $\Theta := \{\alpha_1, ..., \alpha_\xi\}$; and (3) m diverting agents are inserted into the flock. Assuming that the flocking neighbors graph has η connected components, the goal is determining the m diverting agents' initial placements such that, for each desired orientation α_i ($1 \leq i \leq \xi$), there will be at least one flocking agent which will converge to this orientation.

Definition 5.3. **MDIP** (**M**inimal **D**iverting agents **I**nitial **P**lacements) - The input is as follows: (1) The initial placements of the k flocking agents; and (2) The desired orientations after convergence: $\Theta := \{\alpha_1, ..., \alpha_\xi\}$. Assuming that the flocking neighbors graph has η connected components, the goal is determining the minimal number of diverting agents and their initial placements such that, for each desired orientation α_i ($1 \leq i \leq \xi$), there will be at least one flocking agent which will converge to this orientation.

It should be noted that the **DIP** and **MDIP** problems are dealing with the convergence of **at least one flocking agent** to each desired orientation α_i ($1 \leq i \leq \xi$). This stems from the fact that we are not considering the maximization of the disagreement measure, but we are willing to at least reach the minimum disagreement possible (reach some lower bound on the disagreement), which might be sub-optimal. Thus, we define the following two problems:

Definition 5.4. **DIP-MAX** and **MDIP-MAX** - Identical to the **DIP** and **MDIP** problems (respectively), except for the fact that we are also aiming to maximize the disagreement measure.

6 Polynomial Time Complexity - Required Properties

Following [3], the diverting agents are assumed to have a *Face Desired Orientation behavior*, which is sufficient for guaranteeing consensus on a desired orientation, while employing them into a flocking model that is based on the Vicsek Model [29]. Accordingly, given that $\eta \geq \xi$, in Subsection 6.1 we prove that the **DIP** and **MDIP** problems are polynomial in time. In Subsection 6.2, we discuss a case in which the **DIP-MAX** and **MDIP-MAX** problems are also polynomial in time. We prove that, for any fixed $\xi \geq 2$ (the number of desired orientations), those problems can be solved in $O(\eta k^\xi)$ time.

6.1 DIP and MDIP

In this section, we will be showing that the **DIP** and **MDIP** problems are both **polynomial** in time, given that $\eta \geq \xi$. In order to relate the clustering problem to the **DIP** problem we make the following definitions (according to Shamir et al. [21]):

Definition 6.1. (*Cluster Editing/Completion/Deletion Set*) If $G = (V, E)$ is any graph and $F \subseteq V \times V$ is such that $G' = (V, E \Delta F)$ is a cluster graph, then F is called a *cluster editing set* for G ($E \Delta F$ denotes the symmetric difference between E and F, i.e., $(E - F) \cup (F - E)$). If in addition $F \cap E = \emptyset$, then F is called a *cluster completion set* for G. If $F \subseteq E$, then F is called a *cluster deletion set* for G. If G' is a ξ-cluster graph, then F is called a *ξ-cluster editing/completion/deletion set* for G. We denote by $P(F)$ the partition of V according to F.

Hence, the literal meaning of the size of a cluster completion set is as follows: *it counts the number of edge operations (both additions and deletions) needed to transform a graph into a cluster graph.* In the case of the **ξ-CLUSTER-COMPLETION** problem [21], it counts the number of edge additions only. Thus, regarding Definition 5.1, the **DIP** problem becomes a variant of the **ξ-CLUSTER-COMPLETION** problem. Let us consider the following definition.

Definition 6.2. (*The Distance Between Connected Components*) Given a flocking neighbors graph at time step 0, $\mathcal{G}(0) = (\mathcal{V}, \mathcal{E}(0))$, and a pair of connected components C_i, C_j in $\mathcal{G}(0)$, let $d(C_i, C_j) := \min_{v \in C_i, u \in C_j} ||p_v(0) - p_u(0)||$ denote the distance between them.

The following lemma shows how many diverting agents are required for joining together two distinct connected components in the flocking neighbors graph at time step 0 into a single coalition. Furthermore, it gives us lower and upper bounds on the size of the optimal completion set required for such a partition.

Lemma 6.3. *Let C_i, C_j be a pair of distinct connected components in $\mathcal{G}(0)$ ($C_i \cap C_j = \emptyset$). Therefore, if $d(C_i, C_j) \leq 2R$, then a **single** diverting agent should be inserted for connecting between them. Otherwise, **two** diverting agents a_{q_i}, a_{q_j} are required, which are initially placed in the neighborhood of a flocking agent corresponding to C_i, C_j (respectively). In both cases, at least 4 edges and at most $2k \cdot m_{i,j}$ edges are added to the flocking neighbors graph, where $m_{i,j}$ is the required number of diverting agents.*

Proof. Since $C_i \cap C_j = \emptyset$, then $d(C_i, C_j) > R$. Otherwise, there is a pair of vertices $v \in C_i, u \in C_j$ for which $d(C_i, C_j) = ||p_v(0) - p_u(0)|| \leq R$. Therefore: $a_v \in N_u(0)$, that is $(v, u) \in E(0)$ according to the definition of a flocking neighbors graph. This means that $u, v \in C_i \cap C_j$ holds - which is a contradiction. Thus, the proof is divided into two cases. If $R < d(C_i, C_j) \leq 2R$, there is a pair of vertices $v \in C_i, u \in C_j$ for which $R < d(C_i, C_j) = ||p_v(0) - p_u(0)|| \leq 2R$. Thus, inserting a single diverting agent $a_q \in A^D$ into the intersection of the neighborhoods of the flocking agents a_u, a_v will create the desired connection. Moreover, this will result in adding at least 4 edges to the flocking neighbors graph at time step 0, since the following edges will be necessarily added: $(v, q), (q, v), (u, q), (q, u)$. Similarly, if the diverting agent will lie in the neighborhood of more flocking agents, then at most $2k$ edges will be added.

If $d(C_i, C_j) > 2R$, according to Definition 6.2, for each pair of vertices $v \in C_i, u \in C_j$, it holds that $||p_v(0) - p_u(0)|| > 2R$, resulting in $Disc_v(0) \cap Disc_u(0) = \emptyset$. Therefore, **two** diverting agents a_{q_i}, a_{q_j} are required, which are initially placed in the neighborhood of a flocking agent corresponding to C_i, C_j (respectively). Similarly to the previous case, each one will add at least 2 edges and at most $2k$ edges to the flocking neighbors graph. □

Therefore, the following problem is equivalent to the **DIP** problem given that $\eta \geq \xi$:

Definition 6.4. **DIP-CLUSTER-COMPLETION** - The input is as follows: **(1)** A set of desired orientations after convergence - $\Theta := \{\alpha_1, ..., \alpha_\xi\}$; **(2)** The flocking neighbors graph at time step t,

Algorithm 1 DIP-CLUSTER-COMPLETION

1: $S_0 = \{\{\{\emptyset\}, \ldots, \{\emptyset\}\}\}$
2: **for** i = 1 to η **do**
3: $\quad S_i = \{v[j].\textbf{add}(C_i) | v \in S_{i-1}, 1 \leq j \leq \xi\}$
4: Pick in S_η a vector v^* minimizing:

$$s(v) := 2k \sum_{j=1,|v[j]|-1>1}^{\xi} \sum_{i=1}^{|v[j]|-1} [d(v[j]_i, v[j]_{i+1}) \leq 2R?1:2] \quad (4)$$

$\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t))$, with $\eta \geq \xi$ connected components during time step 0; and **(3)** m diverting agents. The goal is finding a partition $P = (V_1, \ldots, V_\xi)$ of the graph $\mathcal{G}(0)$, for which the size of the ξ-cluster completion set implied by P is at most $2km$.

The following theorem proves that using Algorithm 1, the problem is solvable in polynomial time.

Theorem 6.5. *Let $\xi \geq 2$ be fixed. Then, the **DIP-CLUSTER-COMPLETION** problem can be solved in $O(\eta k^\xi)$ time.*

Proof. We will now be adjusting the algorithm given by Shamir et al. [21] to our problem. Let $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t))$ be the flocking neighbors graph. Clearly: $|\mathcal{V}| = k$. Let η be the number of connected components of $\mathcal{G}(0)$ (can be done using either BFS or DFS in $O(|\mathcal{V}|+|\mathcal{E}(0)|)$ time). To find the optimum completion set, we compute partitions of the η components of $\mathcal{G}(0)$ into ξ sets (splitting no connected components). As in [21], using dynamic programming, we only need to consider a polynomial number of partitions.

Let $C_1, \ldots, C_\eta$ be the connected components in $\mathcal{G}(0)$. In contrast to [21], each connected component is also characterized by its location in $\mathbb{R}^2$. Therefore, we shouldn't consider **all** possible partitions, but only those which take into consideration the euclidean distance between each pair of connected components. Without loss of generality, we assume that the connected components are sorted, using $d(\cdot, \cdot)$ as a comparator (Definition 6.2), in a descending order (in the worst case, this can be done in $O(\eta^2)$ time).

In contrast to [21], Algorithm 1 denotes each possible partition by a ξ-sized set of sets S_i of the sets, which correspond to all possible partitions of $C_1, \ldots, C_i$. S_i's j-th set comprises of all connected components generating the j-th cluster. We assume there is no order upon the elements in each such set, meaning that we also don't allow duplicate elements in each set. For instance, if $\xi = 2$, then the set $\{\{C_i\}, \{C_j\}\}$ is identical to $\{\{C_j\}, \{C_i\}\}$. Hence, given a partition, regarding Lemma 6.3, we seek to minimize the **maximal** number of edges possible, which can be added to the flocking neighbors graph at time step 0. Note that $v[j].add(C_i)$ in line 3 of the algorithm denotes the addition of C_i as an element of the set $v[j]$ and $|v[j]|$ denotes the cardinality of $v[j]$. For brevity, the conditional expression in Equation 4 is utilized, whereas it equals to 1 if and only if $d(v[j]_i, v[j]_{i+1}) \leq 2R$. Otherwise, it equals to 2. As illustrated in the supplementary material [5], a set corresponding to some cluster in the partition might contain the empty set $\emptyset$, thus requiring the convention that $d(C_i, \emptyset) := R$ for each connected component C_i in $\mathcal{G}(0)$.

Let v^* be the vector returned by Algorithm 1, which regards the connected components' spatial position, and let F^* be the implied ξ-cluster completion set. Lemma 6.3 then provides us with v^*'s geometric relation, characterizing the set of diverting agents' initial positions with respect to the resulting partition. Hence, similarly to [21], F^* is optimal. In light of Definition 6.1, if $|F^*| \leq 2km$, then F^* is a solution for the **DIP-CLUSTER-COMPLETION** problem. Otherwise, more diverting agents are required.

For the algorithm's time complexity, note that the **for** loop in line 2 iterates η times. In line 3, we iterate $O(k)$ elements ξ times. At total, we have a time complexity of $O(\eta k^\xi)$. □

Since the **DIP-CLUSTER-COMPLETION** problem is equivalent to the **DIP** problem from a graph theoretic point of view, the **DIP** problem is also solved in $O(\eta k^\xi)$ time, given that $\eta \geq \xi$. Note that the algorithm above also solves the **MDIP** problem in $O(\eta k^\xi)$ time according to the following corollary:

Corollary 6.6. *Let $\xi \geq 2$ be fixed. Assuming that the flocking neighbors graph has $\eta \geq \xi$ connected components during time step 0, Algorithm 1 solves the **MDIP** problem in $O(\eta k^\xi)$ time and provides the minimal number of diverting agents required, $m_{min}(\xi)$.*

Proof. Let v^* be the vector returned by the algorithm and let F^* be the implied ξ-cluster completion set. Similarly to [21] and according to Lemma 6.3, it is optimal. Therefore, in light of Definition 6.1, for every ξ-cluster completion set F: $|F^*| \leq |F|$. Therefore, according to Lemma 6.3, the minimal number of diverting agents required is given by $\frac{|F^*|}{2k}$, that is:

$$m_{min}(\xi) = \sum_{j=1,|v^*[j]|-1>1}^{\xi} \sum_{i=1}^{|v^*[j]|-1} [d(v^*[j]_i, v^*[j]_{i+1}) \leq 2R?1:2]$$

□

6.2 DIP-MAX and MDIP-MAX

According to Theorem 4.4, when also considering the maximization of the disagreement measure, the problem becomes **NP-complete**. Regarding Equation 2, given that $P = (V_1, \ldots, V_\xi)$ is the resulting partition of the flocking neighbors graph at time step 0, supposing r is the integral quotient and q is the remainder when k is divided by ξ, so that $k = r\xi + q$, then the maximum value of the disagreement measure is obtained when $|V_i| = r$ for $1 \leq i \leq \xi - q$ and $|V_i| = r + 1$ for $\xi - q + 1 \leq i \leq \xi$. Hence, Algorithm 1 can be restricted to all partitions satisfying this property. If such a partition exists, it can thus be obtained in polynomial by **only aggregating connected components**. Otherwise, the general case arises, according to which such a partition requires a physical separation within several connected components, so as to achieve the desired cardinality of each coalition V_i.

6.3 The Optimal Number of Desired Orientations

Given a static number of agents, determining the number of desired orientations ξ_{OPT} which will lead to the maximal disagreement possible is **NP-hard** due to Theorem 4.4. The following corollary is a direct outcome of Lemma 3.4, which gives us lower and upper bounds on this desired number of coalitions.

Corollary 6.7. *Given k flocking agents and m diverting agents, the number of desired orientations ξ_{OPT} which will lead to the maximal disagreement possible satisfies:*

$$D_{max}(2) \leq D_{max}(\xi_{OPT}) \leq D_{max}(\min(m,k))$$

Following Corollary 6.6, the following corollary considers a scenario in which ξ_{OPT} can be calculated in polynomial time.

Corollary 6.8. *Let η be the number of connected components in the flocking neighbors graph at time step 0. Given k flocking agents and $m \leq \eta$ diverting agents, the number of desired orientations which will lead to the maximal disagreement possible is $\xi_{OPT} = m$ if and only if $m \geq m_{min}(m)$.*

7 Experiments

In this section, we give a concise subset of our experiments, testing the behavior of the diverting agents, which should eventually lead to a spatial consensus-prevention in the observed flock in both a fixed topology and a switching topology. Results regarding the CPU time and the Runtime appear in the supplementary material [5].

7.1 Simulation Environment

We situate our research within the Flockers domain of the MASON simulator [15]. This simulator encodes all the dynamics as they are described in the previous sections, where each agent points and moves in the direction of its current velocity vector. We made a few alterations to the MASON Flockers domain, such that they will fit our needs. It was initially altered to also contain *diverting agents*. Another modification was making the *flocking agents* update their orientation according to the average orientation of all agents in $N_i(t)$ (including itself) at time step t. For more realistic implications of the simulator, its toroidal feature was removed. That is, if an agent moves off of an edge of our domain, it will not reappear and will remain *"lost"* forever. All code alterations are provided in [4].

7.2 Placement Methods

For guaranteeing that the *flocking* neighbors graph does indeed consist of a specific number of connected components, we consider the *grid placement method* and the *random placement method* proposed in [3], according to which each pair of successive flocking agents are within a radius of at most R from each other. Throughout both placement methods, for each connected component C_i, we calculate the maximal and minimal coordinates at which flocking agents are initially located with respect to both the x-axis and the y-axis, which we denote by $x_i^{min}, x_i^{max}, y_i^{min}, y_i^{max}$ (denoted by $\mathcal{B}_i$). For ensuring that two adjacent connected components C_i, C_{i+1} are indeed distinct, we enforce that $x_{i+1}^{min} - x_i^{max} > R$. Regarding the *random placement method*, it should be noted that the flocking agents might be initially spread across $\mathcal{B}_i$ in a *high* density, thus occupying a *smaller* area within $\mathcal{B}_i$. Hence, there are more positions at which placing a diverting agent won't influence any of the flocking agents, resulting in a *slower* convergence rate. In contrast, in the *grid placement method* they are initially within an exact distance of $R-1$, thus entirely occupying the area within $\mathcal{B}_i$. Indeed, regarding the experimental results which follow, it can be observed that the *grid placement method* performs *better* than the *random placement method*.

Determining the "best" initial placements of the diverting agents is **NP-hard** [10]. Hence, for influencing a cluster comprising of the connected components $C_{i_1}, \ldots, C_{i_j}$, each diverting agent is initially placed randomly within the rectangular box formed by $\min_\ell x_{i_\ell}^{min}, \max_\ell x_{i_\ell}^{max}, \max_\ell y_{i_\ell}^{min}, \min_\ell y_{i_\ell}^{max}$, for the sake of increasing the probability at which it will indeed influence them all and solely them. As mentioned earlier, aggregating connected components is also required. Following the *Intersection Points Placement method* proposed in [3], given a pair of connected components C_i, C_j such that $d(C_i, C_j) \leq 2R$, for a random pair $u \in C_i, v \in C_j$ which satisfies $||p_u(0) - p_v(0)|| \leq 2R$, a diverting agent a_q is initially placed randomly along the linear line, connecting the intersection points of both flocking agents' neighborhoods.

7.3 "Lost" Agents

Genter [10] considered cases in which some flocking agents may become indefinitely separated from a flock with a switching topology. Hence, each "lost" agent is considered as a cluster on its own. For a fixed ξ, we denote by $k_i^e(\xi), \ell_i^e(\xi)$ the number of agents and the total number of "lost" agents in the i^{th} cluster during execution e (respectively). Thus, the disagreement measure in Equation 1 is altered as follows (while denoting $\ell^e(\xi) := \sum_{1 \leq i \leq \xi} \ell_i^e(\xi)$):

$$D_{lost}^e(\xi) = \frac{k^2 - \sum_{1 \leq i \leq \xi}(k_i^e(\xi) - \ell_i^e(\xi))^2 - \ell^e(\xi)}{k^2 - k} \tag{5}$$

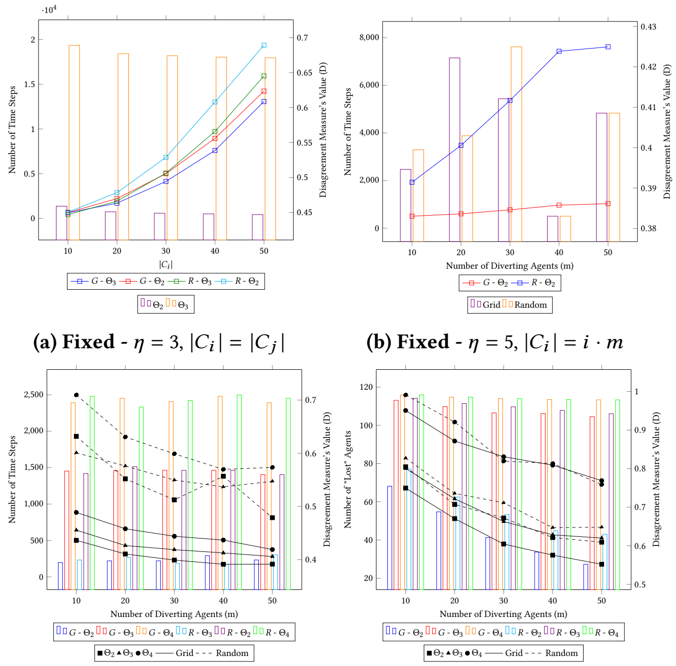
The following lemma proves that $D_{lost}^e(\xi)$ is **strictly decreasing** as a function of $\ell^e(\xi)$ (proof is omitted due to space constraints, and can be found in the supplementary material [5]).

Lemma 7.1. *Let ξ be the number of desired orientations. For a pair of executions $e_1 < e_2$, such that $\ell^{e_1}(\xi) < \ell^{e_2}(\xi), k_i^{e_1}(\xi) = k_i^{e_2}(\xi)$, the following holds: $D_{lost}^{e_1}(\xi) > D_{lost}^{e_2}(\xi)$.*

7.4 Experimental Setup

The baseline settings for variables are as follows: domain height and width: 500 units, agent velocity $v_i = 0.2$ units/sec, and visibility range $R = 10$ units. Flocking agents are initially placed with random initial headings throughout the domain. In our experiments, we conclude that a coalition has converged to an orientation α when every agent (that is not a diverting agent) is facing within 0.01 radians of α. Other stopping criteria, such as when 90% of the agents are facing within 0.01 radians of α, could have also been used. Moreover, due to the involvement of randomness in our simulations, each point in all graphs corresponds to the average over 100 consecutive executions.

All of the experiments incorporate either $\Theta_2 := \{\alpha_1 = \pi, \alpha_2 = 0\}$, $\Theta_3 := \{\alpha_3 = \pi, \alpha_4 = \frac{\pi}{2}, \alpha_5 = 0\}$ or $\Theta_4 := \{\alpha_6 = \frac{\pi}{4}, \alpha_7 = \frac{3\pi}{4}, \alpha_8 = \frac{5\pi}{4}, \alpha_9 = \frac{7\pi}{4}\}$ as the set of desired orientations ($\xi \in \{2,3,4\}$). Furthermore, the experimental results which are introduced consider consensus-prevention among a flocking neighbors graph with $\eta \geq \xi$ connected components, $C_1, \ldots, C_\eta$, for both the fixed topology case and the switching topology case. C_i, C_{i+1} are initialized to be within $1.5R$ from each other ($d(C_i, C_{i+1}) = 1.5R$). Besides, we enforce $y_i^{min} = y_j^{min}$, for the sake of enlarging the area at which a diverting agent can be initially placed in regard with Subsection 7.2. Following



(a) **Fixed - $\eta = 3$, $|C_i| = |C_j|$**

(b) **Fixed - $\eta = 5$, $|C_i| = i \cdot m$**

(c) **Fixed - $\eta = 5$, $|C_i| = i \cdot 10$**

(d) **Switching - $\eta = 5$, $|C_i| = i \cdot 10$**

Figure 1: Comparison for both the *fixed topology* case (1a-1c) and the *switching topology* case (1d), with respect to two parameters for each Θ_j: the *time steps* and the *disagreement measure's value* (bar charts). The values of $\eta, \xi, |C_i|$ are explicitly provided underneath each graph. The placement methods are as they are presented in Subsection 7.2, where G and R stand for *Grid* and *Random* (respectively).

[3], it is sufficient that each diverting agent **constantly** orients itself to the desired orientation. For each one of the presented graphs, we consider both the *grid placement method* or the *random placement method* described earlier. Our main interest is investigating the impact of the number of diverting agents on the disagreement measure in different scenarios, as well as the number of time steps required until reaching the desired consensus-prevention.

7.5 Experimental Results

In Fig. 1a the desired partition regarding Θ_2 is $P = (C_0, C_1 \cup C_2)$, and we solely incorporate j diverting agents with respect to Θ_j, which is the *minimal* number of diverting agents required. As expected, the *higher* the number of desired orientations, the *better* the performance. Theoretically, the task of aggregating C_1, C_2, while using only a *single* diverting agent a_{i+k}, results in a_{i+k} influencing *double* the number of flocking agents influenced by it, when considering Θ_3 as the set of desired orientations. Hence, the number of time steps required until convergence *increases* as a function of ξ. Hence, utilizing the *minimal* number of diverting agents does indeed lead to the desired consensus-prevention, but adding *more* diverting agents yields a better performance (in terms of convergence time).

Regarding Lemma 6.6 and Corollary 6.6, $m_{min}(2) = m_{min}(3) = 3$, $m_{min}(4) = 4$. Hence, in Figs. 1b,1c, with respect to Θ_j, at each execution we first randomly choose a suitable partition P that requires $m_{min}(j)$ of diverting agents, which we then utilize to aggregate connected components for achieving P. The remaining diverting agents are spread randomly between the resulting clusters. As observed by Fig. 1b, the time steps *increase* as a function of m, in a rate which is *approximately linear.* The anomaly in D's value at $m = 40$ stems from the fact that both k and m are varied, meaning that the disagreement measure can't be calculated in advance. Regarding Fig. 1c, it can also be observed that the *time steps increase* as a function of ξ. Given that k is constant, each cluster can contain at most $\lfloor \frac{k}{\xi} \rfloor$ flocking agents, a magnitude which *decreases* as a function of ξ. Thus, each flocking agent individually will be influenced (directly or indirectly) by *fewer* agents, resulting in a *slower* convergence. Furthermore, for each Θ_j separately, the convergence rate *increases* as a function of m, as expected. We also note that the anomaly at $m = 40$ arises since there are executions in which one cluster might contain more agents, and thus take longer to converge.

Regarding Subsection 7.3, Fig. 1d considers the same scenario for *a switching topology*, with the exception that it considers the *number of "lost" agents* instead of considering the time steps required for convergence. As expected, as the number of diverting agents *increases*, the number of "lost" agents *decreases*. Indeed, inserting more diverting agents into the flock will result in more flocking agents being influenced (directly or indirectly) to orient towards the desired orientation, thus leading to a consensus-prevention. However, for each Θ_j, it seems as if *increasing* the number of diverting agents has an opposite impact on the disagreement measure's value, which *decreases*. This was to be expected due to Lemma 7.1.

8 Conclusions and Future Work

We have formally introduced the *consensus-prevention problem*, concentrating on guaranteeing that the swarm will never converge to the same direction by the use of *diverting agents*. We proposed a disagreement measure within a flock, and gave a correlation between our problem and the coalition formation game, according to which we proved that maximizing the disagreement measure is equivalent to maximizing the coalition structure's payoff, making the general problem of maximizing the disagreement in a swarm NP-hard. Accordingly, we have analyzed cases in which the problem is solvable in polynomial time, when the number of connected components in the flocking neighbors graph at time step 0 is *at least* the number of desired orientations after convergence ($\eta \geq \xi$). Finally, we demonstrated in simulation the impact of the number of diverting agents on the disagreement measure in different scenarios, as well as the limitations of such agents in dynamic settings.

Future work warrants examination of the complementary case ($\eta < \xi$), in which a physical separation (edge **deletions**) within several connected components is required, so as to achieve the desired consensus-prevention using approximation and heuristic methods. In such cases we speculate that complex behaviors are necessary to guarantee consensus-prevention, thus extra focus will be given to that. Finally, we also intend to consider the concept of disagreement when it reaches its extreme, where each flocking agent disagrees with all the other flocking agents.

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