Priced Gerrymandering

Extended Abstract

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ABSTRACT

We initiate the study of the gerrymandering problem when changing the district of a voter incurs a certain cost. In this problem, the input is a set of voters having votes over a set of alternatives, a graph on the voters, a partition of voters into connected districts, a cost of every voter for changing her district, a budget, and a target winner. We need to compute if the given partition can be modified so that (i) the target alternative wins the resulting election, (ii) the modification is budget feasible, and (iii) every new district is connected. We study four natural variants of the above problem – the graph on the voters being arbitrary vs complete graph (corresponds to removing the connectivity requirement for districts) and the cost of moving every voter being uniform vs non-uniform. We show that all the four problems are NP-complete even under quite restrictive scenarios. Hence, our results show that district based elections are quite resistant under this new kind of electoral attack. We complement our intractability results by showing that two of our problems admit polynomial-time algorithms if the budget or the number of districts is a constant.

KEYWORDS

Gerrymandering; control; bribery; manipulation; social choice; voting; algorithm; graph

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1 INTRODUCTION

A fundamental problem in multiagent systems is to aggregate preferences of a set of agents into a societal preference. Voting has served as one of the important tools for this aggregation task in various applications (see for example [4, 24]). We assume that agents or voters simply vote for an alternative to express their preferences. The plurality voting protocol is arguably the most widely used voting system where the winners are the set of alternatives who receive a maximum number of votes. In this paper we focus on a district based election system. In such system, the voters are partitioned into districts. The winner of the election is the alternative who wins in the maximum number of districts. Indeed many real world election systems follow this model: the electoral college in US presidential elections, Indian political election, etc. are important examples of use of district based elections in practice.

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However, a typical voting system can come under various kinds of election control attacks — a set of agents, either internal (e.g. voters) or external (e.g. briber), may be able to successfully swing the outcome of the election in their favor. We refer to [15] for an overview of common control attacks on voting systems. Bartholdi et al. [2] initiated the study of computational complexity of various election control problems and since then it has been a major research focus in computational social choice (see [6, and references therein] for example). Bartholdi et al. [2] and Hemaspaandra et al. [17] studied the computational complexity of an important control problem namely *"Control by Partitioning Voters into Two Districts."* This fundamental problem has recently been generalized along two dimensions — (i) the number of districts can be any integer k which is given as input, (ii) there is a graph on the set of voters and every district is required to be a connected subgraph of this graph. This problem is called *gerrymandering* [5, 22]. Indeed there have been serious allegations that some political parties in the US effectively manipulated some elections in their favor through gerrymandering [11, 18].

Lev and Lewenberg [21] observed that, although districts being connected is a fundamental requirement for various district based election scenarios like political election, in some other applications, the connectedness constraint is irrelevant. Examples of such applications include election within an organization, election performed over an online platform, etc. Lev and Lewenberg called this problem *reverse gerrymandering*.

Motivation. All the existing work on election control by voter partition and gerrymandering study the problem of designing a partition from scratch — the input is a set of voters and one needs to find a partition favoring some alternative. However, in typical applications of this type of election control, district based political election for example, there already exists a partition of voters into districts and it may not be feasible for someone to change the existing partition too much. In particular, even if there exists a partition $\mathcal{P}$ of the voters into districts where a target alternative wins the election, constructing that partition $\mathcal{P}$ from the existing partition Q may require changing the district of too many voters which makes $\mathcal{P}$ infeasible. Also the effort/cost required for moving a voter from one district to another may depend on the voter and the pair of districts involved. For some voters, it may be infeasible to change her current district. We incorporate these requirements into four computational problems and provide an extensive complexity landscape of these problems.

Contribution. In our most general problem which we call $Gerrymandering, we are given a partition $\mathcal{P}$ of the voters, a graph $\mathcal{G}$ on the voters, a cost function π which specifies the cost of moving any voter from a district to another, a target alternative c, and a

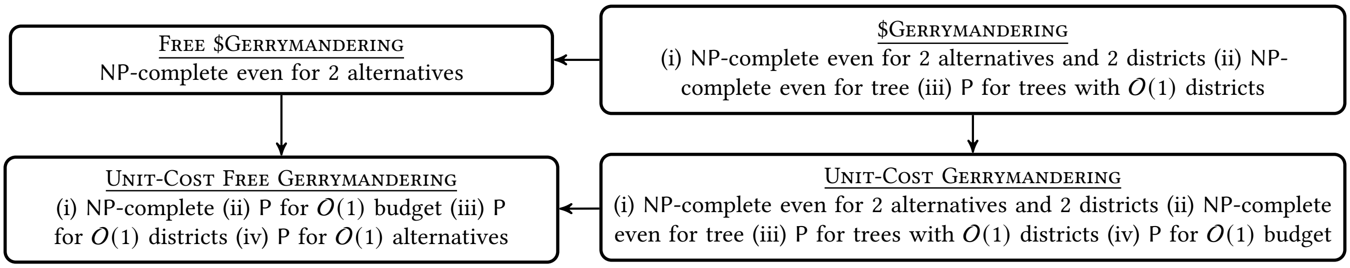

Figure 1: Summary of results and complexity theoretic relationship among problems studied in paper. For two problems X and Y, we write $X \rightarrow Y$ to denote that problem Y many-to-one reduces to problem X in polynomial-time.

budget $\mathcal{B}$. We need to compute if there exists another partition Q of voters into connected districts which is budget feasible and makes c a plurality winner in the maximum number of districts. We show that the $Gerrymandering problem is NP-complete even if we have only 2 alternatives and the graph on the voters is bipartite. However, if the graph on the voters happens to be a tree and the number of districts is only constant, then we show that there exists a polynomial-time algorithm for the $Gerrymandering problem.

Motivated by the concept of reverse gerrymandering, we define and study the Free $Gerrymandering problem which is the same as the $Gerrymandering problem except there is no graph on the voters and consequently there is no requirement for districts to be connected the voters are "free" to move from one district to another of course subject to their costs. It seems that the existence of a graph on the voters may not be the main reason for $Gerrymandering to be intractable since we show that the Free $Gerrymandering problem too is NP-complete even if we have only 2 alternatives.

We study both the $Gerrymandering and Free $Gerrymandering problems under the assumption that the cost of every transfer is 1. We call these problems Unit-Cost Gerrymandering and Unit-Cost Free Gerrymandering respectively. These two problems also capture the robustness of a partition. We call a partition *robust* if many voters need to change their current district to change the winner of the election. We show that the Unit-Cost Free Gerrymandering problem is NP-complete in general but polynomial-time solvable if the budget is a constant, or the number of districts is a constant or we have a constant number of alternatives. On the other hand, the Unit-Cost Gerrymandering problem turns out to be much harder: it is NP-complete even if we have only 2 alternatives and 2 districts. We also show that Unit-Cost Gerrymandering is NP-complete even if the graph is a tree. We summarize our contribution in this paper in Figure 1.

We believe that the novelty of our work lies in bringing together two well-studied topics in computational social choice namely (i) gerrymandering/partitioning voters and (ii) differential cost of actions (bribery for example). We provide a comprehensive complexity landscape of four fundamental computational problems that arise quite naturally here.

Related Work. To the best of our knowledge, Vickrey [28] was the first to coin the idea of automated re-districting. We refer to the excellent survey material by Altman [1] for a comprehensive overview of the early development of automated re-districting. Bartholdi et al. [2] are the first to formally study, among other types of election control, the computational complexity of the problem of making a favorite alternative win by partitioning the voters into two districts. Subsequently, Hemaspaandra et al. [17] studied extensively both the constructive and destructive version of this problem under two tie breaking rule — tie promote (TP) and tie eliminate (TE). Miasko and Faliszewski [23] showed that some election control problems, which are otherwise tractable, becomes NP-hard if we introduce price. Lewenberg et al. [22] introduced the gerrymandering problem and showed that gerrymandering is NP-complete for election systems where voters in each district first elect a representative and the elected representatives ultimately choose a winner. In our model, voters directly vote for the set of candidates fighting an election. Cohen-Zemach et al. [5] showed that gerrymandering is NP-complete for district based election system. Erdèlyi et al. [10] studied the election control problem by partitioning voters subject to various constraints like the size of each district should be almost equal, partitioning into more than two districts, some specified subsets of voters must be placed together, etc. Ito et al. [19] extensively studied algorithmic aspects of gerrymandering for different graph classes. Lev and Lewenberg [21] studied iterated dynamics which reach to a stable equilibrium in the context of reverse gerrymandering where voters change their districts driven by their self interest [21]. van Bevern et al. [27] generalized the gerrymandering problem to a network-based vertex dissolution problem in a graph theoretic setting and showed a dichotomy between tractable and intractable cases of this problem. Puppe and Tasnádi [25] introduced a notion of fairness in the context of gerrymandering and showed that the corresponding computational problem is NP-complete. Puppe and Tasnádi [26] provided an axiomatic study for gerrymandering. Gupta et al. [16] show that the gerrymandering problem with weighted voters on paths is NP-complete but fixed parameter tractable with respect to the number of districts as the parameter. Matthias et al. [3] provide dichotomy results for the complexity of gerrymandering on paths and cycles.

A related phenomenon is bribery where an external agent, called briber, pays the voters to change/misreport their preference so that a preferred alternative of the briber wins the election. Depending on the pricing model of the voters, various notions of bribery have been studied. Prominent models among these variants of bribery include $bribery, shift bribery [12–14], swap bribery [9], safe bribery [20], frugal bribery [8], local distance constrained bribery [7], etc. We refer to [15] for an excellent overview of various bribery problems studied in computational social choice.

REFERENCES

[1] Micah Altman. 1997. Is automation the answer: The computational complexity of automated redistricting. *Rutgers Computer & Tech. L.J.* 23 (1997).

[2] John J Bartholdi III, Craig A Tovey, and Michael A Trick. 1992. How hard is it to control an election? *Math Comput Model* 16, 8-9 (1992), 27–40.

[3] Matthias Bentert, Tomohiro Koana, and Rolf Niedermeier. 2021. The Complexity of Gerrymandering Over Graphs: Paths and Trees. *arXiv preprint arXiv:2102.08905* (2021).

[4] Abhijnan Chakraborty, Gourab K. Patro, Niloy Ganguly, Krishna P. Gummadi, and Patrick Loiseau. 2019. Equality of Voice: Towards Fair Representation in Crowdsourced Top-K Recommendations. In *Proc. 2nd ACM Conference on Fairness, Accountability, and Transparency, FAT*. 129–138.

[5] Amittai Cohen-Zemach, Yoad Lewenberg, and Jeffrey S. Rosenschein. 2018. Gerrymandering Over Graphs. In *Proc. 17th International Conference on Autonomous Agents and MultiAgent Systems, AAMAS*. 274–282.

[6] Vincent Conitzer and Toby Walsh. 2016. Barriers to Manipulation in Voting. In *Handbook of Computational Social Choice*. 127–145.

[7] Palash Dey. 2021. Local distance constrained bribery in voting. *Theor. Comput. Sci.* 849 (2021), 1–21. https://doi.org/10.1016/j.tcs.2020.10.005

[8] Palash Dey, Neeldhara Misra, and Y. Narahari. 2017. Frugal bribery in voting. *Theor. Comput. Sci.* 676 (2017), 15–32. https://doi.org/10.1016/j.tcs.2017.02.031

[9] Edith Elkind, Piotr Faliszewski, and Arkadii Slinko. 2009. Swap bribery. In *Proc. 2nd International Symposium on Algorithmic Game Theory (SAGT 2009)*. Springer, 299–310.

[10] Gábor Erdélyi, Edith Hemaspaandra, and Lane A. Hemaspaandra. 2015. More Natural Models of Electoral Control by Partition. In *Algorithmic Decision Theory - 4th International Conference, ADT*. 396–413.

[11] Robert S Erikson. 1972. Malapportionment, gerrymandering, and party fortunes in congressional elections. *Am. Political Sci. Rev.* 66, 4 (1972), 1234–1245.

[12] Piotr Faliszewski. 2008. Nonuniform bribery. In *Proc. 7th International Conference on Autonomous Agents and Multiagent Systems (AAMAS)*. International Foundation for Autonomous Agents and Multiagent Systems, 1569–1572.

[13] Piotr Faliszewski, Edith Hemaspaandra, and Lane A Hemaspaandra. 2006. The complexity of bribery in elections. In *Proc. 21st International Conference on Artificial Intelligence (AAAI)*, Vol. 6. 641–646.

[14] Piotr Faliszewski, Edith Hemaspaandra, and Lane A Hemaspaandra. 2009. How hard is bribery in elections? *J. Artif. Intell. Res.* 35, 2 (2009), 485–532.

[15] Piotr Faliszewski and Jörg Rothe. 2016. Control and Bribery in Voting. In *Handbook of Computational Social Choice*. 146–168.

[16] Sushmita Gupta, Pallavi Jain, Fahad Panolan, Sanjukta Roy, and Saket Saurabh. 2021. Gerrymandering on graphs: Computational complexity and parameterized algorithms. In *International Symposium on Algorithmic Game Theory*. Springer, 140–155.

[17] Edith Hemaspaandra, Lane A. Hemaspaandra, and Jörg Rothe. 2007. Anyone but him: The complexity of precluding an alternative. *Artif. Intell.* 171, 5-6 (2007), 255–285.

[18] Samuel Issacharoff. 2002. Gerrymandering and political cartels. *Harv. L. Rev.* 116 (2002), 593–648.

[19] Takehiro Ito, Naoyuki Kamiyama, Yusuke Kobayashi, and Yoshio Okamoto. 2019. Algorithms for Gerrymandering over Graphs. In *Proc. 18th International Conference on Autonomous Agents and MultiAgent Systems (AAMAS)*. 1413–1421.

[20] Neel Karia, Faraaz Mallick, and Palash Dey. 2022. How Hard is Safe Bribery?. In *Proc. 121st Conference on Autonomous Agents and MultiAgent Systems, AAMAS*.

[21] Omer Lev and Yoad Lewenberg. 2019. Reverse Gerrymandering: Manipulation in Multi-Group Decision Making. In *Proc. 33rd Conference on Artificial Intelligence, AAAI*. 2069–2076.

[22] Yoad Lewenberg, Omer Lev, and Jeffrey S. Rosenschein. 2017. Divide and Conquer: Using Geographic Manipulation to Win District-Based Elections. In *Proc. 16th Conference on Autonomous Agents and MultiAgent Systems, AAMAS*. 624–632.

[23] Tomasz Miasko and Piotr Faliszewski. 2016. The complexity of priced control in elections. *Ann Math Artif Intel* 77, 3-4 (2016), 225–250.

[24] David M. Pennock, Eric Horvitz, and C. Lee Giles. 2000. Social Choice Theory and Recommender Systems: Analysis of the Axiomatic Foundations of Collaborative Filtering. In *Proc. 17th National Conference on Artificial Intelligence (AAAI)*. 729–734.

[25] Clemens Puppe and Attila Tasnádi. 2008. A computational approach to unbiased districting. *Math Comput Model* 48, 9-10 (2008), 1455–1460.

[26] Clemens Puppe and Attila Tasnádi. 2015. Axiomatic districting. *Soc. Choice Welf.* 44, 1 (2015), 31–50.

[27] Rene Van Bevern, Robert Bredereck, Jiehua Chen, Vincent Froese, Rolf Niedermeier, and Gerhard J Woeginger. 2015. Network-based vertex dissolution. *SIAM J. Discrete Math.* 29, 2 (2015), 888–914.

[28] William Vickrey. 1961. Counterspeculation, auctions, and competitive sealed tenders. *J. Financ. Econ.* 16, 1 (1961), 8–37.