Popularity and Strict Popularity in Altruistic Hedonic Games and Minimum-Based Altruistic Hedonic Games

Extended Abstract

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ABSTRACT

We consider average-based and min-based altruistic hedonic games and study the problem of verifying popular and strictly popular coalition structures. While strict popularity verification has been shown to be coNP-complete in min-based altruistic hedonic games, this problem has been open for equal-treatment and altruistic-treatment average-based altruistic hedonic games. We solve these two open cases of strict popularity verification and then provide the first complexity results for popularity verification in (both average- and min-based) altruistic hedonic games, where we cover all three degrees of altruism.

KEYWORDS

Cooperative Game Theory; Coalition Formation; Hedonic Game; Popularity; Altruism; Computational Complexity

ACM Reference Format:


1 INTRODUCTION

Much work has been done in recent years to study hedonic games, coalition formation games where players express their preferences over those coalitions that contain them. Drèze and Greenberg [9] were the first to propose hedonic games and Bogomolnaia and Jackson [4] and Banerjee et al. [3] formally defined and investigated them. For more background and the rich literature on hedonic games, we refer to the book chapters by Aziz and Savani [2] and Elkind and Rothe [10] and the survey by Woeginger [20].

We focus on altruistic hedonic games (AHGs) that, based on the friend-and-enemy encoding of the players’ preferences due to Dimitrov et al. [8], were introduced by Nguyen et al. [16]. Schlueter and Goldsmith [18] generalized them to “super AHGs,” using ideas of the “social distance games” due to Bränzei and Larsson [6], Bullinger and Kober [7] introduced the related notion of loyalty in hedonic games. Nguyen et al. [16] defined three degrees of altruism depending on the order in which players take their own or their friends’ preferences into account. They chose to model players’ utilities by taking the average of these friends’ valuations in the same coalition. Wiechers and Rothe [19] studied the same three degrees of altruism for minimum-based utilities, and Kerkmann and Rothe [15] applied the original model to coalition formation games in general. For an overview of various notions of altruism in both cooperative and non-cooperative game theory, we refer to the survey by Rothe [17].

We study both average-based and min-based AHGs. For these two classes of games (and for hedonic games in general), many stability notions have been studied, including stability based on single-player deviations (such as Nash stability) or on deviations by groups of players (such as core stability) (see, e.g., [2, 10, 20]).

By contrast, for popularity and strict popularity we look at entire coalition structures (i.e., partitions of the players into coalitions) and ask—similarly to the notion of (weak) Condorcet winner in voting—whether a (strict) majority of players prefer a given coalition structure to every other coalition structure. Previous literature on popularity in hedonic games is, e.g., due to Aziz et al. [1], Brandt and Bullinger [5], and Kerkmann et al. [13]. We study the complexity of the problem of verifying (strictly) popular coalition structures in AHGs. While strict popularity verification is known to be coNP-complete in all three degrees of min-based AHGs [19] and for so-called selfish-first average-based AHGs [16], its complexity was open for the other two degrees of average-based altruism.

We solve these two missing cases via technically rather involved constructions in Section 3. In addition, in Section 4 we provide the first complexity results for popularity verification in average-based and min-based AHGs, covering for both all three degrees of altruism. We show that the problem in all cases is coNP-complete.
by a network of friends, an undirected graph where two players are connected by an edge if and only if they are friends of each other.

Nguyen et al. [16] introduce altruism into a player $i$’s preference by incorporating the valuations of those friends of $i$’s that are in the same coalition into $i$’s utility, considering the average of these friends’ valuations. Wiechers and Rothe [19] vary this model by considering the minimum of those friends’ valuations instead. For any coalition $A \in N^j$, we use the following notations:

\[
\begin{align*}
\text{avg}_i^F(A) &= \frac{1}{|A|^2} \sum_{a \in A \cap F_i} v_a(A) \\
\min_i^F(A) &= \min_{a \in A \cap F_i} v_a(A) \\
\max_i^F(A) &= \max_{a \in A \cap F_i} v_a(A)
\end{align*}
\]

We now define popularity, which is based on the pairwise comparison of coalition structures. For a hedonic game $(N, \succeq)$ and two coalition structures $\Gamma, \Delta \in \mathcal{C}_N$, let $\mathcal{P}_{\Gamma \rightarrow \Delta} = \{ i \in N \mid \Gamma \succ_i \Delta \}$ be the number of players that prefer $\Gamma$ to $\Delta$. A coalition structure $\Gamma \in \mathcal{C}_N$ is popular (respectively, strictly popular) if, for every other coalition structure $\Delta \in \mathcal{C}_N$, $\Delta \neq \Gamma$, it holds that $|\mathcal{P}_{\Gamma \rightarrow \Delta}| \geq |\mathcal{P}_{\Delta \rightarrow \Gamma}|$ (respectively, $|\mathcal{P}_{\Gamma \rightarrow \Delta}| > |\mathcal{P}_{\Delta \rightarrow \Gamma}|$).

The min-based altruistic preferences, denoted by $\minSF$ (respectively, $\minAF$), are defined analogously, using the minimum instead of the average. A pair $(N, \succeq)$, where $\succeq$ is a profile of preferences defined by one of the average-based degrees of altruism, is called an altruistic hedonic game (AHG) with average-based altruistic preferences $\succeq$. A game $(N, \preceq)$ with min-based altruistic preferences $\preceq$ is said to be a min-based altruistic hedonic game (MBAHG).

We have solved the two remaining open problems regarding the complexity of strict popularity verification in AHGs, namely for equal treatment and altruistic treatment (Theorem 1). The proofs to establish the same results for SP-Exi are inspired by a proof by Wiechers and Rothe [19, Thm. 4].

Theorem 3. $P$-Veri is coNP-complete for SF AHGs and MBAHGs.

Finally, we turn to P-Exi. Note that we cannot simply modify the proofs of the preceding theorems in order to show the hardness of P-Exi (similarly to how we used Theorem 1 to obtain Corollary 2) because a tie between two most popular coalition structures would not suffice to show the nonexistence of a popular coalition structure. However, for both AHGs and MBAHGs and in all three degrees of altruism, there exist examples where no popular coalition structures exist, and we suspect that P-Exi is hard for all considered models.

5 CONCLUSIONS

We have solved the two remaining open problems regarding the complexity of strict popularity verification in AHGs, namely for equal treatment and altruistic treatment (Theorem 1). The proofs of these results required new ideas and are technically demanding. In addition, we have provided the first complexity results for popularity verification in AHGs and MBAHGs, covering both all three degrees of altruism (Theorems 3, 4, and 5). Hence, the complexity of popularity verification and strict popularity verification is now settled in AHGs and MBAHGs; the picture is complete.

ACKNOWLEDGMENTS

We thank the reviewers. This work was supported in part by DFG grant RO 1202/21-1 and the NRW project “Online Participation.”
REFERENCES


