We study strategic behavior of project proposers in the context of participatory budgeting. We assume that the votes are fixed and known and the proposers want to set as high project prices as possible, provided that their projects get selected and the prices are not below the minimum costs of their delivery. We study the existence of Nash equilibria in such games. Furthermore, we report an experimental study of the games we propose.

KEYWORDS
Participatory Budget; Equilibria; Cost Selection; Game Theory


1 INTRODUCTION
Consider a certain city that wants to use participatory budgeting [2, 3, 7] to let its inhabitants decide what improvements to implement. The city council fixed the available budget and asked people to submit their ideas. Specifically, each citizen could submit a project where he or she would outline the type of action to take as well as the cost of carrying it out. The citizens quickly seized the opportunity and came up with a number of proposals. However, they also realized that choosing the costs of the projects is not obvious. For example, to find the cost of building a bike path one might ask a construction company for a quote, but one would get a whole range of costs, depending on the width of the path, the materials used, the possible adaptations of the surrounding area, and so on. Indeed, the more expensive a project is, the better it fulfills its goals, but also the less likely it is to be funded (for example, due to a limited budget). Our approach is to analyze the strategic nature of project cost selection under various participatory budgeting rules.

The Game. We assume that the sets of projects and of voters, who indicate which projects they approve, are fixed. Each project is controlled by a different proposer choosing its cost so that it is as high as possible while remaining selected. However, each project also has the lowest cost under which it can be reasonably implemented and the proposers prefer costs that are at least as high. Importantly, whether a voter approves a project or not, does not depend on its cost. The projects are chosen according to a given rule. In other words, we consider a game where project proposers (or, for simplicity, the projects) are the players, project costs are their strategies, and costs of selected projects (minus their delivery costs) are their payoffs. We analyze whether these games have pure Nash equilibria and, if so, what costs are reported under these equilibria.

2 PRELIMINARIES
Participatory Budgeting. We define a PB instance as a tuple \( E = (P, V, B, \text{cost}) \), where \( P = \{p_1, \ldots, p_m\} \) is a set of projects, \( V = \{v_1, \ldots, v_n\} \) is a set of voters, \( B \in \mathbb{R}_+ \) is the available budget, and \( \text{cost}: P \to \mathbb{R}_+ \) is a function specifying the cost of each project.

BasicAV. It starts with \( W = \emptyset \) and considers all the projects following their nonincreasing approval scores (with ties broken using \( > \)), inserting a considered project \( p \) into \( W \) if \( \text{cost}(W \cup \{p\}) \leq B \).

AV/Cost. It is like BasicAV, but orders projects by \( |A(p)|/\text{cost}(p) \).

Phragmén [1, 4]. Phragmén starts with \( W = \emptyset \). Initially, the voters have empty accounts and they continuously earn money at the same pace. When there is a project \( p \) whose voters have \( \text{cost}(p) \) funds and is within the remaining budget, \( p \) is included in \( W \), the accounts of voters in \( A(p) \) are set to zero, and \( p \) is removed from consideration. If \( \text{cost}(W \cup \{p\}) > B \), then \( p \) is removed from consideration. At a time, the best such project in \( > \) is taken. The rule outputs \( W \) when all projects are removed from consideration.

Method of Equal Shares (MES-Cost) [5, 6]. First, each voter receives \( B/|V| \) amount of money. Then, we let \( W = \emptyset \). Within...
We provide a theoretical and an experimental study of our games. We investigated, for a given voting rule, the equilibrium costs. Under BasicAV, as expected, all the budget goes to the most popular project. Under AV/Cost, every project submits a cost proportional to its support. For BasicAV, AV/Cost, and MES-Cost/Ph most of the projects "reached" the costs predicted by the NE. Our conclusion from this experiment is that under AV/Cost or Phragmén the proposers are incentivized to use costs that reflect their approval score. Under MES-Cost/Ph, most strongly supported projects and those that are supported by many voters disapproving more popular projects can request higher costs. So, in an NE, MES-Cost/Ph funds fewer projects than AV/Cost or Phragmén.

3 RESULTS

We provide a theoretical and an experimental study of our games.

Theoretical Results. We investigated, for a given voting rule, if an NE exists for every PB game. While BasicAV and, surprisingly, MES-Cost always admit an equilibrium, it is not true for the other rules we consider. Our results are dependent on the delivery costs and on specific tie-breaking orders. As such, for AV/Cost or Phragmén with plurality ballots, an NE is guaranteed for sufficiently low delivery costs, but for higher ones, it ceases not to exist for some tie-breaking orders. In fact, for Phragmén, an NE might not exist for arbitrary delivery costs. Our theoretical results are shown in Table 1.

Experimental Results. Our goal was to compute (approximate) equilibria under our rules. While for BasicAV or AV/Cost we could compute the equilibria using our theoretical results, this would not be possible for Phragmén. Hence, we simulate the following dynamics (where we could compute an NE directly our simulations gave nearly identical results, so we expect that the results are also meaningful for the other rules). First, proposers report the cost that was originally chosen for their project. Then, in each iteration, one of them, selected uniformly at random, increases or decreases their project's cost. Specifically, the proposer chooses a number \( x \) in \([0, \text{cost}/10]\) uniformly at random (where \( \text{cost} \) is the current project's cost) and changes their project's cost by \( x \) if it increases their utility. We expect to converge to an NE if one exists.

In Figure 1 we show the results of the dynamics after 10 000 iterations. Under BasicAV, as expected, all the budget goes to the most popular project. Under AV/Cost, every project submits a cost proportional to its support. For BasicAV, AV/Cost, and MES-Cost/Ph most of the projects "reached" the costs predicted by the NE.

Our conclusion from this experiment is that under AV/Cost or Phragmén the proposers are incentivized to use costs that reflect their approval score. Under MES-Cost/Ph, most strongly supported projects and those that are supported by many voters disapproving more popular projects can request higher costs. So, in an NE, MES-Cost/Ph funds fewer projects than AV/Cost or Phragmén.

4 CONCLUSION

We have introduced a game-theoretic model capturing strategic cost selection in PB. In future work, one could, e.g., consider dropping the assumption that project proposers have full knowledge of ballots, or study the complexity of checking the existence of an NE.

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