Decision Making in Non-Stationary Environments with Policy-Augmented Search

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ABSTRACT

Sequential decision-making is challenging in non-stationary environments, where the environment in which an agent operates can change over time. Policies learned before execution become stale when the environment changes, and relearning takes time and computational effort. Online search, on the other hand, can return sub-optimal actions when there are limitations on allowed runtime. In this paper, we introduce Policy-Augmented Monte Carlo tree search (PA-MCTS), which combines action-value estimates from an out-of-date policy with an online search using an up-to-date model of the environment. We prove several theoretical results about PA-MCTS. We also compare and contrast our approach with AlphaZero, another hybrid planning approach, and Deep Q Learning on several OpenAI Gym environments and show that PA-MCTS outperforms these baselines.

KEYWORDS

Sequential Decision-Making; Non-Stationary Environments; MCTS

1 INTRODUCTION

Sequential decision-making is present in many important problem domains, such as autonomous driving [2], emergency response [7], and medical diagnosis [1]. An open challenge in such settings is non-stationary environments, where the dynamics of the environment can change over time. A decision agent must adapt to these changes to avoid taking sub-optimal actions. Reinforcement learning (RL), especially with deep neural networks, struggles in such settings due to quickly outdated policies and the high cost of retraining [4, 8], while Monte Carlo tree search (MCTS) offers quicker adaptation but faces challenges with slow convergence in complex situations. This can be especially problematic in time-sensitive contexts [9], leading to potentially delayed responses [5]. In this work, we present a novel hybrid decision-making approach called Policy-Augmented Monte Carlo tree search (PA-MCTS), which combines a policy’s action-value estimates with the returns generated by MCTS without changing either of the two approaches, i.e., the combination occurs entirely outside the online search tree. We argue that a hybrid decision-making approach that integrates RL and online planning can combine their strengths while mitigating their weaknesses in non-stationary environments. The intuition is that if the environment has not changed too much between when an optimal policy was learned and when a decision needs to be made, the policy can still provide useful information for decision-making. We also show how existing hybrid approaches, e.g., AlphaZero, can also be used for decision-making in non-stationary environments.

2 MARKOV DECISION PROCESSES IN NON-STATIONARY SETTINGS

Our focus is on scenarios where it’s impractical to immediately learn a new policy after changes in the environment are detected, aiming to optimize decision-making during the transition to learning a new, nearly optimal policy. We explore this through the lens of non-stationary Markov decision processes (NSMDP), which introduce a temporal dimension to stationary MDPs and assume changes in the transition function are smoothly bounded [6]. Recognizing that some changes can be abrupt and significant, we propose a variant called transition-bounded non-stationary Markov decision processes (T-NSMDP), which limits the overall shift in
transition probabilities between the initial learning phase and execution, to better manage the adaptability of decision-making processes in dynamic environments. Consider the transition probability function \( P_t(s' | s, a) \), where the subscript \( t \) denotes the time step under consideration. Now, consider that the environment undergoes some change between time steps 0 and \( t \). We assume:

\[
\forall s, a : \sum_{s' \in S} P_t(s' | a, s) - P_0(s' | a, s) \leq \eta \quad \text{where} \quad t \in T
\]

(i.e., some point in time after the original policy was learned), and \( \eta \in \mathbb{R}^+ \) is a scalar bound. Although our algorithm only tackles discrete changes for now, our problem definition is agnostic to whether the change is continuous or discrete.

Our key hypothesis is that with small changes in the transition function, the Q function under an optimal policy does not change much, i.e., “good” actions remain valuable, and “bad” actions do not suddenly become promising. We show that the change in \( Q \) is bounded with respect to the change in \( P \):  

**Theorem 2.1.** If \( \forall s, a : \sum_{s' \in S} P_t(s' | a, s) - P_0(s' | a, s) \leq \eta \), and \( \forall s, a : |r(s, a)| \leq R \), and the discount factor \( \gamma < 1 \), then:

\[
|Q_0^*(s, a) - Q_t^*(s, a)| \leq \epsilon \quad \forall s, a, \quad \text{where} \quad \epsilon = \frac{\gamma \eta R}{(1 - \gamma)^2}  \quad \text{(The proof is presented in the arxiv version).} \]

The objective is to identify optimal actions at time \( t \), maximizing future rewards under certain conditions: an initial optimal action-value function \( Q_0^*(s, a) \) learned under different conditions, bounded changes in transition probabilities by \( \eta \), an up-to-date black-box simulator for the current environment, and constrained computational resources preventing the learning of a new optimal policy at execution time.

3 POLICY AUGMENTED MONTE CARLO TREE SEARCH

**Policy-Augmented Monte Carlo Tree Search (PA-MCTS)** integrates an online search with Q-values learned on the environment at an earlier decision epoch, even if the environment has changed. Rather than selecting an action based on the highest expected return estimated by the online search, PA-MCTS instead chooses an action that maximizes a convex combination of the previously learned Q-values and the MCTS estimates \( \overline{G} \):

\[
\arg \max_{a \in A_s} \alpha Q_0^*(s, a) + (1 - \alpha) \overline{G}(s, a) \quad \text{(1)}
\]

where \( Q_0^*(s, a) \) is the optimal \( Q \)-function previously learned by the decision agent. The hyper-parameter \( \alpha \), set between 0 and 1, moderates the balance between learned Q-values and MCTS-derived expected returns, aiming to find a middle ground between low-variance, biased \( Q_0^* \) estimates and high-variance, unbiased \( \overline{G} \) estimates. Below, we quantify the total error in the expected return using PA-MCTS compared to an optimal (updated) policy.

**Theorem 3.1.** When PA-MCTS is used for sequential decision making, the maximum difference between the return from an optimal policy and the return from following PA-MCTS is at most \( \frac{2(\alpha \epsilon - \alpha \delta \delta + \delta)}{1 - \gamma} \).

(The proof is presented in the arxiv version).

4 EXPERIMENTS

Our approach is tested using four OpenAI Gym environments: Cart Pole, Frozen Lake, Cliff Walking, and Lunar Lander [3]. In Cart Pole, we introduce non-stationarity by varying the gravitational constant and the pole’s mass. For Frozen Lake and Cliff Walking, non-stationarity is simulated by adding probabilities for unintended movements. In Lunar Lander, wind is introduced as a non-stationary factor, requiring the agent to adapt its actions to wind force.

Results for partial environments and settings we use to induce non-stationarity are presented in Table 1. The complete results are presented in the arxiv version. Our implementation is available at https://github.com/scope-lab-vu/PAMCTS. We observe as the change in the environment increases, the performance of the DDQN policy in isolation degrades as hypothesized. Second, we observe that PA-MCTS converges significantly faster than standard MCTS (with appropriate \( \alpha \)). Third, in most environment settings, PA-MCTS outperforms Alphazero, Standard MCTS and DDQN.

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Table 1: Results for all four environments with varying levels of non-stationarity. For each environment, the degree of change increases from top to bottom. We observe PA-MCTS comprehensively outperforms baseline approaches, including AlphaZero.

<table>
<thead>
<tr>
<th>Environment</th>
<th>Setting</th>
<th>DDQN</th>
<th>MCTS</th>
<th>AlphaZero</th>
<th>PA-MCTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cartpole (g = 9.8)</td>
<td></td>
<td>2500.0 ± 0.0</td>
<td>846.456 ± 43.228</td>
<td>2403.261 ± 35.946</td>
<td>2500.0 ± 0.0</td>
</tr>
<tr>
<td>(g = 20)</td>
<td></td>
<td>2500.0 ± 0.0</td>
<td>918.022 ± 46.534</td>
<td>2278.90 ± 32.9</td>
<td>2500.0 ± 0.0</td>
</tr>
<tr>
<td>(g = 50)</td>
<td></td>
<td>22.061 ± 0.729</td>
<td>778.511 ± 44.90</td>
<td>1920.261 ± 78.547</td>
<td>2500.0 ± 0.0</td>
</tr>
<tr>
<td>(g = 500)</td>
<td></td>
<td>7.083 ± 0.126</td>
<td>111.578 ± 17.705</td>
<td>626.178 ± 80.091</td>
<td>934.656 ± 90.150</td>
</tr>
<tr>
<td>Frozen Lake</td>
<td>[1.000, 1.000, 1.000]</td>
<td>1.0 ± 0.0</td>
<td>1.0 ± 0.0</td>
<td>1.0 ± 0.0</td>
<td>1.0 ± 0.0</td>
</tr>
<tr>
<td></td>
<td>[0.833, 0.833, 0.833]</td>
<td>0.830 ± 0.012</td>
<td>0.806 ± 0.012</td>
<td>0.809 ± 0.013</td>
<td>0.830 ± 0.012</td>
</tr>
<tr>
<td></td>
<td>[0.633, 0.183, 0.183]</td>
<td>0.522 ± 0.016</td>
<td>0.56 ± 0.017</td>
<td>0.523 ± 0.017</td>
<td>0.587 ± 0.016</td>
</tr>
<tr>
<td></td>
<td>[0.433, 0.283, 0.283]</td>
<td>0.26 ± 0.014</td>
<td>0.764 ± 0.013</td>
<td>0.253 ± 0.014</td>
<td>0.796 ± 0.013</td>
</tr>
<tr>
<td></td>
<td>[0.333, 0.333, 0.333]</td>
<td>0.12 ± 0.01</td>
<td>0.866 ± 0.01</td>
<td>0.114 ± 0.011</td>
<td>0.936 ± 0.009</td>
</tr>
</tbody>
</table>

In principle, we do not require the optimal Q-function. As shown in the experiments, an approximation also works well in practice.
REFERENCES


